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# Learning handwriting with pen-based systems: computational issues

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#### Abstract

This paper introduces the theoretical foundation for the development of a pen-based system dedicated to helping to teach handwriting in primary schools. Knowledge given by a kinematic theory of rapid human movements is used. The system proposed includes a letter model generator which is used to create letter shapes with a human-like kinematics. The system generates feedback to pupils after a multilevel analysis of the handwriting. The analysis presented deals with shape conformity, shape error identification, fluency analysis and kinematic parameter evaluation. Discussion on how fluency measurement and error quantification can be useful in developing a learning metric is also presented. © 2002 Pattern Recognition Society. Published by Elsevier Science Ltd. All rights reserved.

*Keywords:* Handwriting learning; Pen-based computer; Delta-lognormal model; Handwriting generation; Handwriting quality; Handwriting evaluation; Interactive learning tool

## 1. Introduction

In the near future, pen-based computers will provide a means to make writing with a pen as natural as it usually is on paper. Currently, analyzing and structuring an electronic pen trace or a script is still far from being a simple task. Few researchers or companies have explored the idea of using pen-based systems as a tool for teaching handwriting to children [1–3]. Pen-based systems using an integrated LCD display and graphic tablet can capture and display all the input from the digitizer and simulate electronic ink on the computer screen. These capabilities of writing with an electronic pen and seeing the trace, as it would appear on paper, can make the electronic pen-pad a natural hardware tool for teaching handwriting to children. With these kinds of tools, a child can write, with a pen, directly on the surface of the screen without

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the need to raise his head to look at the screen. Thus, to design a global software program capable of executing the principal tasks of a human teacher, the system has to handle the global process dealing with "teaching" from the teacher's point of view, and "learning" from the pupil's point of view. The aim of this complex interactive human process is to transform pupils in school from the status of beginning writers to that of skilled writers. In fact, the human teacher's tasks include many aspects, especially at the kindergarten levels, like adapting exercises, creating enjoyable games for learning, at times paying more attention to some children than others, etc. Only two of the teacher's main tasks are implemented in a pen-based system: (1) Displaying reference handwritten traces of letters on-screen; and (2) Providing relevant feedback to the children about their movements during the writing process, making them conscious of both the correctness and incorrectness of their actions.

Using electronic notepads while showing the handwritten letters dynamically and interactively is of major

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benefit in the teaching process, since the letters are drawn on the screen at the child's request. The sequence of drawing the strokes and the kinematics of the movements can be repeatedly viewed to the child's satisfaction, as compared to other methods of reproducing letters for pupils where only the shapes are presented, along with some basic information about the stroke sequence. Both the shape and the sequence of strokes involved in drawing a letter obey a "ductus", which defines how strokes should be written, and which is usually set by a teaching expert who is responsible for curriculum. The process of showing the kinematics of drawing letters dynamically, based on ink velocity generation, must seem natural in order to be easily reproduced by children.

To provide realistic and relevant feedback about the writing process, the system must be able to analyze and quantify the major errors in the writing process. According to the rules commonly used by the teachers in schools during a handwriting exercise, only the most important errors are to be revealed based on the goal of the current exercise.

In this paper, we present the theoretical foundations of the development of an automated educational pen-based system called "Scriptôt" [4,5] to help teach handwriting in primary schools. The system proposed is also intended to provide a self-learning environment for children between 4 and 7 years old. The basic functionalities of the system ensure that the teacher's two main tasks are fulfilled and other miscellaneous functionalities are included as well. Special care has been taken in the design of the interface, not only to make it simple to use, but also to provide an enjoyable environment for children which will help them discover new ways of learning, and also help them to enjoy handwriting. Children will have their own exercise books, so that they can turn the pages and review their previous work. Many interactive games have also been designed, dealing mainly with skilled pen manipulation. The concept of a "playroom" is proposed, including corners for exploration and discovery [5]. The system features colors, sounds, animation and messages to provide natural feedback.

The first part of the paper presents the fundamentals of the kinematic theory involved in the writing process, showing how a handwriting model can be used to help accomplish the first of the teacher's tasks: generating handwriting and letter references. The next section of the paper deals with teacher's second task: analyzing the child's writing to provide relevant feedback, in terms of both correction and appreciation. Analysis of handwriting is based on two kinds of features: those dealing with the shape aspects of writing and those dealing with movement structure. First, a shape analysis methodology is described, to localize errors in the written letter shape, then the focus shifts to fluency analysis in order to assess handwriting movement, and finally a kinematic analysis is proposed and some results are presented in the form of illustrations, with a discussion on the future directions of the project.

## 2. Fundamentals of handwriting generation

## 2.1. Kinematic theory

The kinematic theory proposed by Plamondon [6–8] is aimed at understanding the generation and control of simple and complex human movements such as hand-writing. In the past few years, this theory has shown its ability to accurately represent and reproduce simple and complex human movements. From simple tapping and pointing movements to cursive handwriting generation, the theory can explain and reproduce the basic characteristics of handwriting in a single framework. But let us first look at the fundamentals of the kinematic theory in the context of handwriting representation and generation.

#### 2.2. Single movement generation

According to the kinematic theory, a single movement can be completely described in the velocity domain, as a response to the synergistic action of a combination of an agonist and an antagonist neuromuscular network [6,7]. Each network of this synergy is represented as a set of complex subsystems, which reacts to an input command  $D_1$  (for the agonist system) and  $D_2$  (for the antagonist system) with a lognormal impulse response. This impulse response  $\Lambda(t; t_0, \mu, \sigma^2)$  can be described by a set of three parameters: the starting time  $t_0$ , the logtime delay  $\mu$  and the logresponse time  $\sigma^2$  [6]. The module of the curvilinear velocity v(t) for a single movement can then be described by the weighted subtraction of the impulse response of the antagonist system from that of the agonist system, resulting in a delta-lognormal velocity profile:

$$v(t) = D_1 \Lambda(t; t_0, \mu_1, \sigma_1^2) - D_2 \Lambda(t; t_0, \mu_2, \sigma_2^2),$$
(1)

where

$$\begin{split} &\Lambda(t;t_0,\mu_i,\sigma_i^2) \\ &= \frac{1}{\sigma_i\sqrt{2\pi}(t-t_0)} \exp\left(\frac{-\left[\ln(t-t_0)-\mu_i\right]^2}{2\sigma_i^2}\right). \end{split}$$

(2)

A single two-dimensional movement or stroke (*i*) can thus be described both in the kinematic and in the static domain by a velocity vector  $\vec{v}(t)$  the magnitude of which is described by a delta-lognormal equation. Each vector starts at time a  $t_{0(i)}$  and position a  $P_{0(i)}$  with an initial direction  $\theta_{0(i)}$  and moves along a circular path of length  $D_{1(i)} - D_{2(i)}$  with a constant curvature  $C_{0(i)}$ . The movement described by this stroke will reach its target with a spatial



Fig. 1. Single stroke.

precision proportional to the ratio of the antagonist and agonist commands  $D_2/D_1$  [6,7] (see Fig. 1).

#### 2.3. Fluent handwriting generation

Using the basic rules of single-stroke generation, fluent handwriting can be seen as a complex movement generation resulting from the spatio-temporal overlap of single strokes described by their delta-lognormal velocity vectors  $\vec{v}_i(t)$  moving along circular paths. The resulting curvilinear velocity vector can then be characterized by its instantaneous magnitude and orientation [8], as described by Eqs. (3) and (4).

$$v(t) = |\vec{v}(t)| = \left|\sum_{i=1}^{n} \vec{v}_i(t - t_{0_i})\right|,$$
(3)

$$\theta(t) = \arctan\left\{\frac{\sum_{i=1}^{n} v_i(t - t_{0_i}) \cos(\theta_i(t - t_{0_i}))}{\sum_{i=1}^{n} v_i(t - t_{0_i}) \sin(\theta_i(t - t_{0_i}))}\right\}.$$
 (4)

Fig. 2 shows an example of a five-stroke movement generated with the model. As can be seen from the

resulting shape of the velocity profile, the number of strokes can be estimated by counting the velocity extrema. The spatio-temporal overlap of the two strokes, (2-3) and (4-5), produces the fluent upper and lower loops of the letter "g" respectively, while discontinuities can be observed between strokes (1-2) and (4-5), resulting in a low velocity magnitude (v(t) nearly 0).

Both the static and the kinematic properties of the resulting movement can be controlled by the parameters of the model, while its fluency is controlled by the time overlap of a stroke relative to its predecessor.

#### 2.4. Letter model generator tool

For the specific needs of our project, handwriting generation software was designed to help the teacher in a classroom. The letter model generator tool is a graphic interface dedicated to the needs of teachers, enabling them to create their own letter models which may be different from the standard models included within the system. The tool is an interactive graphical editor, where strokes are generated dynamically using delta-lognormal velocity profiles from a sketch drawn by the teacher. The letter generator is aimed at providing a dynamic ductus of the letters to be used as models for teaching; and proposes a structural description of the letters needed by the analysis process. The analysis of a child's trace is mainly based on comparing the structure of his or her trace to the model, a process which will be detailed in the next section of the paper.

Designing a letter with this tool requires few steps: first the shape of the letter is processed, then its kinematics is generated according to the delta-lognormal model described above (Eqs. (3) and (4)). The first step



Fig. 2. Stroke overlapping for fluent handwriting.



Fig. 3. Defining letter models.

in the definition of a new letter is to draw a sketch according to a sequence of movements. Fig. 3, column 1, shows some examples of letter generation design. The sequence proposed is considered as the ductus of the new letter model. Each component of the sketch, defined by a pen-down and a pen-up, is converted to a sequence of simple stroke primitives according to the vectorial delta-lognormal model [8,10], as shown in Fig. 3, column 2. The original handwriting shapes and sizes are then edited to fit within the guide-line constraints, see Fig. 3, column 3. The designer can then specify constraints on sizes, shapes, sequences of components, component envelopes relative to guide lines or other components, number of strokes per component, size of strokes, curvatures, etc. The resulting sequences and attributes of the model are then stored to represent the static shape of the letter model, see Fig. 3, column 4.

After the shape of the letter model has been defined, the kinematics of its corresponding movements is generated according to Eqs. (3) and (4). Previous work [9], based on constant velocity model generation, has shown the difficulty children have in following or imitating the movements, mainly because of the artificial nature of constant velocity. Our assumption here is that showing a completely controlled and idealized movement to children can make it easier for them to represent it and to reproduce it.

The model generator tool proposes, then, an idealized handwritten movement with standard parameters. These values can be adjusted from the graphical interface to reproduce any variation of a human-like targeted movement. A set of typical values of parameters can be found in Ref. [10]. The Scriptôt system offers many options for adjusting the velocity of the movement for ease of presentation and reproduction.

## 3. Global handwriting analysis and evaluation

The handwriting analysis module is aimed at measuring errors revealed in the sketch as compared to the model. In other words, the goal of the analysis is to evaluate what makes a letter specimen so different from the expected model. This kind of analysis can be the analysis encountered in typical handwriting recognition problems (see Ref. [11] for an extensive survey).

In our specific learning-monitoring situation, the analvsis has to automatically reflect all the errors or differences between the sketch and the target shape model or its underlying kinematics. Errors or differences have then to be classified in order of importance and relevance, and subsequently presented as appropriate feedback to the child. The relevance of an error reflects, according to the teacher, how important this error is compared to other errors, taking into account the level of dexterity of the child. In the classroom, for example, the teacher usually starts by verifying that a child's handwriting corresponds to the letter he or she has been asked to reproduce. The teacher then pays attention to the following criteria: ductus, overtracing of some strokes, completeness of the letter, position of the letter relative to schoolbook guide-lines, letter size, etc. The analyzer developed follows the same basic concepts, starting with global shape analysis to extract major errors relative to the global letter, followed by component analysis and finally stroke analysis. When the shape is considered with no major shape errors, kinematic analysis is performed.

## 3.1. Shape analysis

The main goal of the shape analysis process is to identify and to localize shape errors on the written specimen when compared to the model. If the written specimen conforms to the expected model, the handwriting analysis process moves to the next step. If the written trace is too different from the target model, no further analysis is needed. Verification of the conformity of a sketch to a model is based on a comparison of the global features of the letter, such as size, x-y ratio and constraints relative to guide lines, then its components, and finally its stroke sequences.

Assuming that M is the reference model and S the sketch produced by a child, both composed of a sequence of attributes  $At_i$  (where 1 *i* Na) and a sequence of components  $Co_i$  (where 1 *i* Nm for the model; and 1 *i* Ns for the sketch), where a component is defined by a pen-down and a pen-up, then:

$$M = [At_1^M, At_2^M, \dots, At_{Na}^M; Co_1^M, Co_2^M, \dots, Co_{Nm}^M],$$
(5)

$$S = [At_1^S, At_2^S, \dots, At_{Na}^S; Co_1^S, Co_2^S, \dots, Co_{Ns}^S].$$
 (6)

Each component  $Co_i$  can also be expressed as a sequence of attributes  $At_{ij}^{Co}$  (where 1 j Nco) and a sequence of stroke primitives  $St_{ij}$  (where 1 j Nst):

$$Co_{i} = [At_{i1}^{Co}, At_{i2}^{Co}, \dots, At_{iNco}^{Co}; St_{i1}, St_{i2}, \dots, St_{iNst}].$$
(7)

Each stroke primitive  $St_{ij}$  is also characterized by a set of attributes  $At_{ijk}^{St}$  (where 1 k Nst), which determines its expected position, length, curvature and angle relative to the global character of the letter, to its components, to guide lines or to other strokes, in addition to a sequence of points  $pt_{ijl}$  (where 1 l Npt).

$$St_{ij} = [At_{ij1}^{St}, At_{ij2}^{St}, \dots, At_{ijNst}^{St}; pt_{ij1}, pt_{ij2}, \dots, pt_{ijNpt}].$$
(8)

*S* is considered similar to *M* if: first, both have similar attributes; second, both have the same number of components (Nm = Ns); third, each pair of components has similar attributes; fourth, each pair of components has the same number of strokes, and, finally, each pair of strokes has similar attributes, as expressed by Eq. (9).

$$M \approx S$$

$$\therefore \begin{cases} At_n^M \approx At_n^S, \\ Co_i^M \approx Co_i^S, \\ \vdots \end{cases} \begin{cases} At_{il}^M \approx At_{il}^S, \\ St_{ij}^M \approx St_{ij}^S, \\ \vdots \end{cases} \{At_{ijk}^M \approx At_{ijk}^S, \end{cases}$$
(9)

where n = 1, ..., Na; i = 1, ..., Nm; l = 1, ..., iNco; j = 1, ..., iNst; k = 1, ..., ijNst; and where  $\approx$  denotes similarity between letters, components, strokes and attributes.

We consider that  $At_i^M$  is similar to  $At_i^S$  if the difference between their value  $E_i$  is less than a threshold T. The value and boundaries of T are fixed experimentally and denote the severity of the similarity criteria. Small boundaries of T require a perfect match between attributes, while large ones allow the system to be more permissive. In practice, the value of the thresholds can be fixed from the teacher's dedicated interface.

When some errors  $E_i$  are greater than the tolerance of T, the system provides feedback to highlight that a particular attribute or portion of the shape does not correspond to that part of the expected model. The system chooses the maximum error among the  $E_i$  and provides corresponding feedback generated by highlighting the error, or by changing the color of the specimen trace that contains the error to red, for example. In that way, the shape errors are better localized and the feedback takes on a graphical significance.

In practice, to make it more flexible, the system can accept instances where Nm is different from Ns. In those cases, when Ns is less than Nm, it means that some components are missing in the written letter. The system generates feedback to the writer requesting completion of the letter. When Ns is greater than Nm, it might mean that some components should be eliminated, and the system can ask the child to delete some extra components.

Greater flexibility can also be reached by allowing verification of the similarity between components that are not necessarily in the same order. Condition (9) then becomes true if, for the allowed combinations of components, the system can find a good similarity match in  $Co_i^S$ . When the allowed correspondence between indices is not respected, the system can provide feedback suggesting that the stroke sequence was not respected.

After seeking out the major differences in shape between the written specimen S and the model M, the system shifts its focus to movement fluency analysis.

## 3.2. Fluency analysis

The measure of fluency tends to express to what extent a handwriting movement has been acquired and effected with no latency, and no delay. The more fluent a movement is, the more at ease the writer is in drawing that sketch. The fluency measure is probably one of the best indicators of the writer's learning behavior. A few simple definitions have been proposed to date [3]. For example, a movement is fluent if its velocity profile presents just one lobe over each stroke. This statement is more of a binary definition of fluency, and indicates whether the movement is fluent or not.

Here, because letter model generation is based on a kinematic theory of rapid human movement where each single stroke movement follows an asymptotically idealized movement through the delta-lognormal relation, we thus consider that the global movement of a letter model represents an idealized movement. Fluency measurement is aimed specifically at the velocity profile of the child movement as compared to the velocity profile of the letter model generator. This comparison is mainly based on computing the quadratic error between the two velocity profiles after proper normalization. The normalizing



Fig. 4. Fluency analysis (a) raw velocity profiles (b) after normalization.

process consists on scaling each profile such that it is within a range of an absolute maximum velocity. Even though we know that there is a nonlinear relationship between the model and the sketch velocities, in this prototype, the velocity profile of the child is scaled linearly over time to reflect similar duration to that of the velocity profile of the model, see Fig. 4.

Fluency is defined by how different the velocity profile  $v_M$  is from velocity profile  $v_S$ . This is expressed by the quadratic mean error between the two velocity profiles

$$E = \frac{1}{T} \sqrt{\sum_{t=0}^{T} (v_M(t) - v_S(t))^2}.$$
 (10)

In the example in the Fig. 4, the trace velocity profile is composed of two lobes, as is the model profile. The smaller the array between the two profiles, the more the movement is considered to be fluent compared to the model's velocity profile.

At the beginning of a training session, the child's movements when reproducing a letter model can be very different from the model's velocity profile, and the error expressed by E, and given in Eq. (10), might be very large. After many training sessions, once the child has achieved some degree of automation in reproducing the letter, this quantity E is reduced. Thus, the evolution of E over time can be considered as a good estimate of the evolution of the writing skills in reproducing the same letter.

#### 3.3. Kinematic analysis

Approaches that deal with the global evaluation of children's handwriting, particularly at a beginner level, were presented in the last sections. However, a kinematic analysis is required to guide the learning process for intermediate users. But, analyzing real human handwriting is not a simple task. It requires an analysis-by-synthesis approach to extract the underlying information pertaining to each single stroke from a real and complex movement [12]. Finding the vector parameters of each single stroke from the measured velocity module is not an easy problem and requires many heuristics and good optimization techniques. Theoretically, there is an infinite number of possible solutions for this problem. In practice, as we limit the search space by simultaneously optimizing the kinematic and the static domains, and by fixing the range of variability for each parameter, the number of solutions is slightly reduced, but can still be high. The optimizing process has to encompass the divergence problem, the estimation of the starting point, which influences the multiple solution problem, and the overall time taken by the algorithm convergence.

For the single movement case, extraction of the set of parameters that best fits a delta-lognormal velocity profile can be achieved in two steps:

- First, estimate the parameters that describe the kinematics of the movement  $D_1$ ,  $D_2$ ,  $\mu_1$ ,  $\mu_2$ ,  $\sigma_1$ ,  $\sigma_2$  and  $t_0$ .
- Second, estimate the static parameters that describe the geometric properties of the movement in the 2D plane, *P*<sub>0</sub>, *C*<sub>0</sub> and θ<sub>0</sub>.

Seeking out the optimal set of kinematic parameters that best fits the observed movement requires the use of robust optimization approaches that ensure algorithm convergence. Knowing the nonlinearity of the delta-lognormal function, a nonlinear regression technique such as the Levenberg–Marquardt technique [13] is used. To ensure convergence, to reduce the search space and to minimize computation time, a graphical method is used to find an approximation of the optimal solution prior to nonlinear optimization [14,15]. To estimate the geometric properties of a single movement, various approaches can be used, and both graphical and analytical methods can be employed to estimate the overall curvature  $C_0$  of a single stroke and its starting angle  $\theta_0$  and starting point  $P_0$  [11].

For the case of complex movement or multiple-stroke movement analysis, such as fluent handwriting, more difficulties can be expected. These are mainly due to the



Fig. 5. (a) Original (continuous lines) and reconstructed (dotted lines) handwriting, (b) original (continuous lines) and reconstructed (dotted lines) curvilinear velocity.

time (s)

spatio-temporal overlap introduced when a fluent movement is decomposed into multiple independent strokes. The analysis of complex movements can thus be divided into five steps [12]:

(b)

- First, estimate the minimal number of strokes (or delta-lognormal curves which compose the curvilinear velocity) that can generate the observed movement.
- Second, estimate the kinematic parameters for each single stroke (*i*): *D*<sub>1(i)</sub>, *D*<sub>2(i)</sub>, μ<sub>1(i)</sub>, μ<sub>2(i)</sub>, σ<sub>1(i)</sub>, σ<sub>2(i)</sub> and *t*<sub>0(i)</sub>, while avoiding, as much as possible, the time-overlapping effects.
- Third, estimate the static parameters that describe the geometric properties of each movement in the 2D plane,  $P_{0(i)}$ ,  $C_{0(i)}$  and  $\theta_{0(i)}$ .
- Fourth, optimize the parameters estimated with nonlinear regression techniques for all the successive vectorial delta-lognormal functions.
- Fifth, make geometrical corrections of parameters,  $P_{0(i)}$ ,  $C_{0(i)}$  and  $\theta_{0(i)}$  to minimize the error between the reproduced shape and the observed shape (and restart step 4 until satisfactory conditions, which can be thresholds on error or on time consumption, are reached).

The analysis proposed here concerning the parameter extraction of each stroke offers a new perspective for the analysis of fluent 2D movements such as handwriting. It shows the hidden part of each individual stroke and the anticipation strategies that can be used to make handwriting fluent: controlling loops, movement length and movement time. Fig. 5 shows a typical parameter extraction result, which decomposes the cursive word "age" into its basic units. Figs. 5(a) and (b) show, respectively, the original handwriting and curvilinear velocity in continuous lines, and the model after extraction and reconstructed with dotted lines. This proposed kinematic analysis offers a new perspective on understanding and evaluating real complex fluent gestures such as letters and cursive handwriting. It decomposes a movement into its fundamental strokes, which can be compared to a target movement for evaluation purposes. The parameters extracted from real handwriting can thus be used to find the best and most realistic stroke generation and anticipation strategies for a group of users that can define what can be considered as an "ideal" movement for a letter, for cursive writing, or even for typical handwriting styles.

## 4. Discussion

The learning tool prototype developed was initially set such that all the child handwriting errors had a predetermined and fixed relevance. In practice, relevance values will have to be adjusted experimentally for each kind of error and will evolve according to the particular learning curve of each child. Experiments to fix these parameters are planned with teachers in many primary schools over the next school year.

As one can expect, children make many more errors as beginners than they usually do after a few training sessions. The fact that the system is able to identify and quantify errors at each level of the movement writing process provides new possibilities for formalizing learning metrics and for monitoring the evolution of the child's learning process over time. This learning process includes two kinds of handwriting skills, as proposed in this study: first, the ability to respect the shape of a letter, producing the so-called "good writing"; and second, the ability to produce fluent movements and gestures respecting a targeted kinematics. Such studies may enlighten us with respect to our understanding of human behavior in terms of shape and pattern representation; influence of age over the learning process; strategies used for achieving fine motor control and comparative studies of training efficiency programs. Results of experimentation in kindergartens and in primary grades were not available for this paper, but will be part of the next step of our project where they are expected to provide new insights into the design of second-generation prototypes.

#### 5. Conclusion

In this paper, we have presented the two main tasks necessary to design a handwriting learning tool using an electronic pen-based environment. The first task of the system is accomplished by a handwriting generator, which shows models of letters from their basic primitives and according to the complex kinematics involved in the pen movements. The second task constitutes the cornerstone of the system, since it enables handwriting analysis, which will generate relevant and immediate feedback for children. This feedback adds a new dimension to the learning process, making the child autonomous and able to take appropriate steps to correct his or her writing movement immediately. Computing relevant feedback is not an easy task, however, and different levels of analysis have been conducted in this study on both the shape and the kinematics of written letters. These include global shape analysis and kinematic shape analysis of handwriting. This analysis will still require future improvements in order to extend its capacities to processing and detecting other kinds of errors, much like human teachers who can detect errors such as overtracing and lines broken by pen-up movements.

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