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Studying the variability of handwriting patterns using the Kinematic Theory

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ABSTRACT

The variability observed in handwriting patterns is analyzed from the perspective of integrating the resulting motor control knowledge in the design of more powerful handwriting recognizers in personal digital assistants (PDAs) and smartphones. Using the highest representational level of the Kinematic Theory of Rapid Human Movement, the Sigma-Lognormal model, this article reports basic theoretical and practical results that could be taken into account in the design of such systems. The main movement variability introduced by the neuromuscular system (NMS) and induced through the scheduling of motor tasks by the central nervous system (CNS) is divided into global and local fluctuations. From a fiducial action plan decoded by this model, a wide range of handwriting distortions are artificially generated by acting on the Sigma-Lognormal parameters. The resulting patterns are studied to understand scale changes and rotational deformations, the two basic features that a recognizer has to take into account. An experiment based on the writing of the same word by six writers is also reported. The results, obtained by an ANOVA analysis, corroborate the predictions and support the relevance of the Kinematic Theory for the analysis and synthesis of handwriting disruptions. These findings consolidate the results of previous studies on single strokes using the Sigma-Lognormal model. Overall, this report provides new insights into our understanding of motor control, as well as into practical cues for the development of huge databases of letters and words to train and test on-line handwriting classifiers and recognizers.

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1. Introduction

One of the key applications of handwriting as a human/computer interface will be its integration into personal digital assistants (PDAs) and smartphones. So far, numerous research projects have been conducted to design such systems (Jäger, Liu, & Nakagawa, 2003; Liu, Jäger, & Nakagawa, 2004), and some studies have suggested to incorporate knowledge of handwriting variability into their design (Plamondon & Srihari, 2000). For example, starting with few specimens of a single word produced by a PDA user, an engineer would like to evaluate the writer's variability and then tune and incrementally adapt a recognizer to a particular handwriting style. So far, most of the effort expended to achieve this has been concerned mainly with the use of mathematical deformation models having almost no direct link to the basics of human motor control (Matic, Guyon, Denker, & Vapnik, 1993; Mouchère, Anquetil, & Ragot, 2005; Nakamura, 2004; Vuori, Aksela, Laaksonen, & Oja, 2000).

To efficiently perform such an integration, preliminary studies, such as modeling of handwriting variability, or defining practical limits for using such a model in PDA applications, are a prerequisite for any motor control-based design.

This article constitutes the very first step in such a research program. It investigates how the Kinematic Theory could be used to define a working framework for the integration of some basic motor control concepts into the blueprint for these new intelligent interfaces. Specially, this study explains how the handwriting variability can be analyzed through the variability of the Sigma-Lognormal parameters.

2. Background knowledge

Probably the most widely accepted observation in goal-directed movement is the stereotypical velocity pattern of single strokes. In early studies, the variability observed in stroke and handwriting patterns was considered to result from random processes, and was studied accordingly using statistical tools (Engelbrecht, 2001). Over the years, several psychophysical studies have highlighted the fact that the velocity profile of a rapid movement is strongly stereotypical. It has been reported that the tangential velocity had a bell-shaped profile (Lacquaniti, Terzuolo, & Viviani, 1983), which was considered to be symmetrical in the earliest studies (Morasso, 1981; Morasso, Mussa-Ivaldi, & Ruggiero, 1983), with only slight variability between participants (Miall & Haggard, 1995) and a decrease in variability with practice (Georgopoulos, Kalaska, & Massey, 1981). Since 1993, precise curve fitting results have confirmed the basic asymmetry of this profile (Plamondon, Alimi, Yergeau, & Leclerc, 1993). Furthermore, it has been suggested that the smooth feature of handwriting trajectories may be due to the temporal overlap of successive submovements executed simultaneously (Morasso et al., 1983; Schillings, Meulenbroek, & Thomassen, 1996). The temporal overlap schedule has been used to describe various features of complex pen-tip paths. In such studies, the end-effector kinematics have been described as a summation of the time-shifted velocity profiles of submovements, and it was suggested that, at the level of the motor planning process, the virtual end-effector trajectories are represented by vectorially adding the velocities of the strokes involved in the generation of a handwriting pattern.

Numerous studies have been conducted to model the processes involved in the production of human movements, which can be classified into various categories. Neural network models (Bullock, Grossberg, & Mannes, 1993; Gangadhar, Joseph, & Chakravarthy, 2007; Guenther & Bullock, 1992; Schomaker, 1991) study the emergence of invariants in arm movements, velocity invariance being considered an intrinsic property of the network of differential equations describing the dynamics of the system. For example, the VITE model proposes an original architecture of neural networks, where the volitional commands for skilled movements are controlled by a "GO" signal (Bullock et al., 1993). Gangadhar et al. (2007) exploited the oscillatory model of *X* and *Y* velocity components, initially proposed by Hollerbach (1981), and designed a handwriting stroke generator, in which the velocity is expressed as resulting from the oscillatory activity of a neuromotor network. Dynamic models focus on the mass-spring characteristics of the muscles (Bizzi, 1980; Hollerbach, 1981) or on the changes in equilibrium-points (Feldman, 1986) to comprehend trajectory formation. Generalized motor program models (Carter & Shapiro, 1984; Nihei, 1985) rely on various representations of the action plan

(Meyer, Smith, & Wright, 1982) or on the diverse stochastic properties of strokes (Harris & Wolpert, 1998). Finally, principle-oriented models track the velocity invariance problem with several minimization criteria: minimum time (Enderle & Wolfe, 1987), minimum acceleration (Neilson, 1993), minimum jerk (Flash & Hogan, 1985), minimum snap (Edelman & Flash, 1987), and minimum torque changes (Uno, Suzuki, & Kawato, 1989).

In contrast, the Kinematic Theory (Plamondon, 1995) represents the volitional commands by a Dirac-Impulse occurring at a time t_0 and the neuromuscular system by a network made up of a large number of linear subsystems, where the non-linearities are embedded in the hypothesis of a proportional effect that governs cumulative time delays of the impulse response as measured at the outputs of adjacent subsystems.

These computational models generally provide, directly or indirectly, an analytical expression describing the velocity profile using a set of parameters. They have also been used to suggest some possible origins of the infinite variability observed in real data, which can be expressed by the variability of their parameters. For example, Longstaff and Heath (1997), Longstaff and Heath (1999, Longstaff and Heath (2003) supported the idea that, while biological systems involved in the production of movement are non-linear, slight variations of initial conditions will lead to a wide and even infinite variability of the output, which means that the handwriting velocity profile can be considered as chaotic realizations.

Various methods use the discontinuous representation scheme to describe complex handwriting gestures with the superimposition of strokes (Djioua & Plamondon, 2007; Morasso et al., 1983; Plamondon & Djioua, 2005; Plamondon & Djioua, 2006; Plamondon, Lopresti, Schomaker, & Srihari, 1999; Plamondon & Srihari, 2000). These strokes are regarded as primitives constituting a specific class of rapid human movements from which complex trajectories are built (Giszter, Mussa-Ivaldi, & Bizzi, 1993; Mussa-Ivaldi, Giszter, & Bizzi 1994; Paine & Tani, 2004; Thoroughman & Shadmehr, 2000; Woch, 2006; Woch & Plamondon, 2003; Woch & Plamondon, 2004). Thereafter, the wide variability observed in handwriting patterns can be interpreted as caused both by the intrinsic variability of the individual strokes and by the fluctuations occurring in the time plan of the superimposition process controlled at the CNS level.

Thus, our approach is based on the assumption that a complex pattern of letters, or a word, results from the superposition, with overlapping, of a set of strokes located both in time and in space, and described by the Sigma-Lognormal parameters.

The use of a Kinematic Theory to study the possible origins of both the single stroke and the handwriting deformations has shown that the distortions of word shapes seem very sensitive to slight changes in the corresponding movement time plan, as represented by a sequence of time occurrences $\{t_{0i}\}$, i.e., to write a readable word, the superposition of the strokes must be planned in advance from a previously learned original plan. Moreover, the motor control parameters that affect the stroke directions and amplitudes seem to be less critical, while the neuromuscular parameters seem to have even less influence on the deformations. These studies have shown the existence of direct relationships between the fluctuations of the Sigma-Lognormal parameters and the observed pattern-warping.

3. Overview of the Sigma-Lognormal model

The Sigma-Lognormal model is considered to be the highest level of representation in the family of models supported by the Kinematic Theory (Djioua, 2007; Plamondon & Djioua, 2006). It considers single strokes as primitives from which complex patterns are built. Each primitive[1] has a lognormal velocity profile and a direction profile described by an error function (erf.). The formal expression of the velocity profile $\vec{v}(t)$ of a complex movement is given by:

$$\vec{v}(t) = \sum_{i=1}^{L} \vec{v}_i(t); \quad L \geqslant 2, \tag{1}$$

[1] The notion of primitive used in the present study differs from that used in studies dealing with the Delta-Lognormal model, since in the latter a stroke has a Delta-Lognormal velocity profile (Woch, 2006; Woch & Plamondon, 2003; Woch & Plamondon, 2004). In the Sigma-Lognormal approach, a stroke has a single lognormal velocity profile, and is thus more basic than the Delta-Lognormal stroke, which is made up of two opposing lognormals.

where L represents the number of strokes involved in the generation of a given pattern and $v_i(t)$ is the velocity profile of the ith stroke.

In 2D space, the x–y Cartesian coordinates of the trajectory are given by:

$$x(t) = x_0 + \sum_{i=1}^{L} \int_{t_{0i}}^{t} v_i(\tau)\cos[\varphi_i(\tau)]d\tau; \quad y(t) = y_0 + \sum_{i=1}^{L} \int_{t_{0i}}^{t} v_i(\tau)\sin[\varphi_i(\tau)]d\tau, \tag{2}$$

with

$$v_i(t) = \frac{D_i}{\sigma_i(t-t_{0i})\sqrt{2\pi}} e^{-\frac{1}{2\sigma_i^2}[\ln(t-t_{0i})-\mu_i]^2}; \quad \varphi_i(t) = \theta_{di} + \frac{(\theta_{fi}-\theta_{di})}{2}\left[1 + erf\left(\frac{\ln(t-t_{0i})-\mu_i}{\sigma_i\sqrt{2}}\right)\right]. \tag{3}$$

Each curved stroke, indexed by i, is completely described by a parameter vector $P_i = \{t_{0i}, D_i, \theta_{di}, \theta_{fi}, \mu_i, \sigma_i\}$ made up of six Sigma-Lognormal parameters, which reflects both the motor control process and the neuromuscular response. Indeed, from a functional point of view, to produce the ith stroke, volitional commands are modeled by a Dirac-Impulse signal occurring at a time stamp t_{0i} and sent to the input of a neuromuscular network modeled by a convolution of a large number of linear subsystems. The impulse command embeds the space features of a stroke, i.e., its length D_i, its starting direction angle θ_{di} and its ending direction angle θ_{fi}. The lognormal impulse response of the neuromuscular system is characterized by the logtime delay μ_i and the logresponse time σ_i.

3.1. Generation of complex patterns

The equations described above can be used to generate any complex handwriting trajectory from the vectorial superimposition of a set of strokes, individually described by a parameter vector P_i (Guerfali & Plamondon, 1995; Plamondon & Guerfali, 1998; Plamondon, Feng, & Woch, 2003; Varga, Kilchhofer, & Bunke, 2005). Fig. 1a illustrates a typical example, where the trajectory of the word "lune" has been modeled by an action plan made up of the concatenation of 14 curved strokes. In this study, 13 strokes are sufficient to model such a cursive word. However, at the end of the movement, a supplementary stroke produced by an antagonist movement appears when subjects stop their end-effector displacement. The Kinematic Theory has underlined this point and proposed to model a velocity profile of a rapid movement with a Delta-Lognormal. This modeling is then applied in the generation of complex trajectories such as handwriting.

As can be seen, each stroke is identified by a start-point and an end-point. According to this virtual target concept (Bullock et al., 1993; Plamondon & Privitera, 1995), the end-point of the ith stroke is concatenated to the start-point of the ith + 1 stroke. From such a discontinuous action plan, the resulting handwriting pattern is generated by activating the control parameters (the amplitude D_i and the direction θ_{di}, θ_{fi}) of each successive stroke, according to the time plan $\{t_{0i}\}$ of their occurrence.

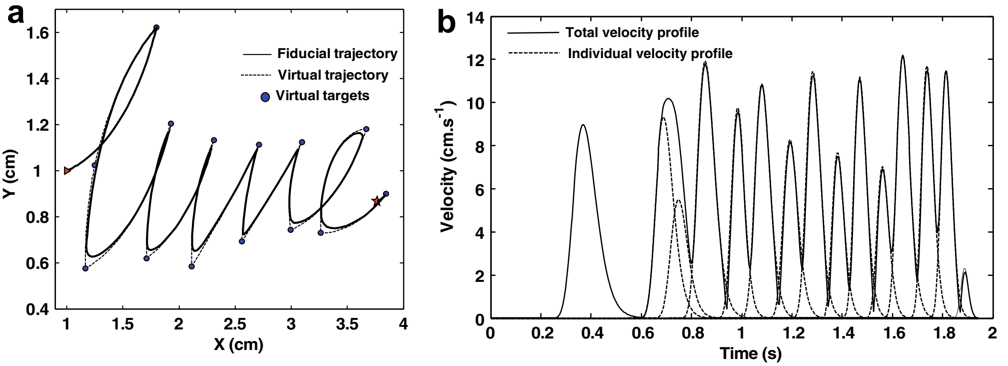

Fig. 1. A typical modeling with Sigma-Lognormal parameters of: (a) the handwriting pattern of "lune"; and (b) its corresponding velocity profile. Note that the word has been built with 14 strokes, 13 of which represent the main pattern and the last stroke depicting the antagonist neuromuscular effect that appears at each end of a movement.

4. Possible origins of pattern variability

In handwriting, various kinds of distortions can be observed in the pattern of a letter or a word. In this investigation, we assume the existence of two principal origins of variability. The first, which is called global variability, affects all the superimposed strokes in the same way. The second, referred to as local variability, represents the independent and intrinsic variability of each individual stroke. [In this study, other sources of variability, for example the one that affects the number of strokes that can be observed in different productions of a pattern, are not considered.]

These fluctuations of the control $(t_{0i}, D_i, \theta_{di}, \theta_{fi})$ and peripheral (μ_i, σ_i) parameters lead to a wide range of deformations, similar to those observed in real data. So, several predictions can be made concerning stroke variability. In the context of PDA applications, we are mainly concerned with size and orientation changes, considering the small writing surface. To enlarge or reduce the length of a stroke of rank i, an homothetic transformation can be made with ratio k_i on the parameter D_i. Similarly, to carry out a rotation around a starting point, the same positive or negative offset δ_i can be added onto the parameters θ_{di} and θ_{fi}. Although the individual variation of t_{0i} does not affect the pattern of a single stroke i, however its influence is huge when more than two strokes are superimposed. In contrast, the parameters (μ_i, σ_i) produce small deformations on the pattern. To highlight these predictions, let us consider the action plan of the word "lune", depicted in Fig. 1a. The distortions observed in an individual pattern performed by various writers through a set of trials can be interpreted as the effect of the global and local variability of the Sigma-Lognormal parameters around the mean values used in the construction of the fiducial pattern. Thus, from the action plan of this latter, several possible deformations can be created by varying its descriptive parameters:

$$D_i \rightarrow (K \pm k_i)D_i, \tag{4a}$$

$$\theta_{di} \rightarrow \theta_{di} \pm R_d \pm \rho_{di}, \tag{4b}$$

$$\theta_{fi} \rightarrow \theta_{fi} \pm R_f \pm \rho_{fi}, \tag{4c}$$

$$t_{0i} \rightarrow t_{0i} \pm \tau_i, \tag{4d}$$

$$\mu_i \rightarrow \mu_i \pm M \pm \Delta\mu_i, \tag{4e}$$

$$\sigma_i \rightarrow \sigma_i \pm S \pm \Delta\sigma_i, \tag{4f}$$

where the global variability is represented by capital letters and the local variability by lowercase Greek letters. Let us recall that the global variability of t_{0i} does not result in any pattern deformations.

4.1. Predictions

For the global effects, the simulations depicted in Figs. 2a–c show that the individual variability of the parameter D_i affects the zoom of the pattern while preserving its temporal scale, a phenomenon that has been reported regularly since the publication of Denier van der Gon and Thuring's findings in

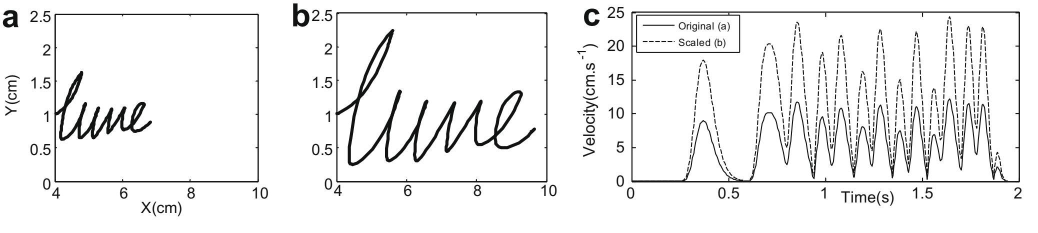

Fig. 2. Global effects, both in the trajectory and the velocity spaces, of a 100% increase ($k_i = 2$) in the parameters D_i. Note that the time scale of the velocity profiles remains stable in this case, i.e., the time scale is preserved.

1965. Rotational effects, as described in Figs. 3a–c, are produced by a global modification of the directional parameters (θ_{di}, θ_{fi}), while keeping the other parameter values constant.

As depicted in Fig. 3c, both the spatial and temporal scales are preserved in this case. However, when the peripheral parameters (μ_i, σ_i) are varied, the theory predicts supplementary smoothing and sharpening effects in the trajectory caused by the overlapping of the lognormal velocity components. Indeed, when the parameters μ_i and σ_i increase, they involve the translation and dilation of the velocity profile of each primitive, causing an increase in the rate of overlapping between adjacent strokes and leading to smoother trajectories (see Fig. 4). In contrast, when these parameters decrease, the overlapping rate decreases and the patterns become sharper (see Fig. 5).

In terms of local variability, the superimposition of independent deformations on each stroke, appearing randomly on local regions of a handwriting trajectory, leads to non-uniform deformations, characterized by a mixing of different levels of scale changes, rotations, smoothing and sharpening. Fig. 6 illustrates local effects resulting from slight variations of all parameters. To perform this trial, the parameter values are randomly chosen inside narrow intervals $[P_i - \Delta P_i, P_i + \Delta P_i]$ centered on the original parameter vector P_i, used to reconstruct a fiducial pattern, while the interval width is fixed by standard deviations ΔP_i of parameter variability, statistically determined from real data.

In real handwriting, global and local variations are mixed, and, by modeling a set of similar patterns with Sigma-Lognormal parameters, these variations on the parameter space can be mapped and analyzed a posteriori, since Eqs. (4a)–(4f) allow the possibility of distinguishing these two kinds of variability. Such an approach is clearly of interest for tuning a recognizer which has already been designed, but what is of interest here is to design an experiment which will allow some of the above predictions to be observed with a view to using this basic knowledge from the start in the design of a recognizer.

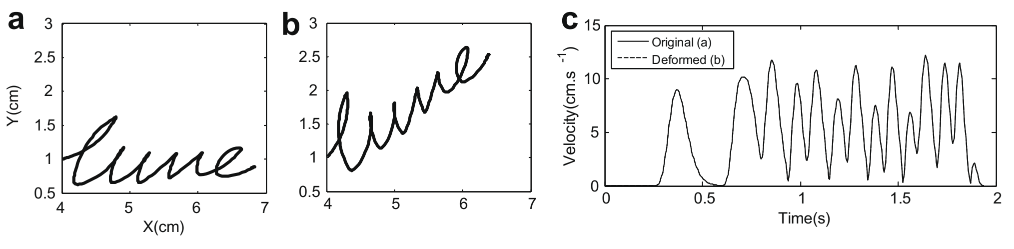

Fig. 3. Global effects, both in the trajectory and the velocity spaces, of a 35° increase in the direction parameters (θ_{di}, θ_{fi}). Note that there are no predicted effects on the velocity profiles, i.e., both the time and space scales are preserved.

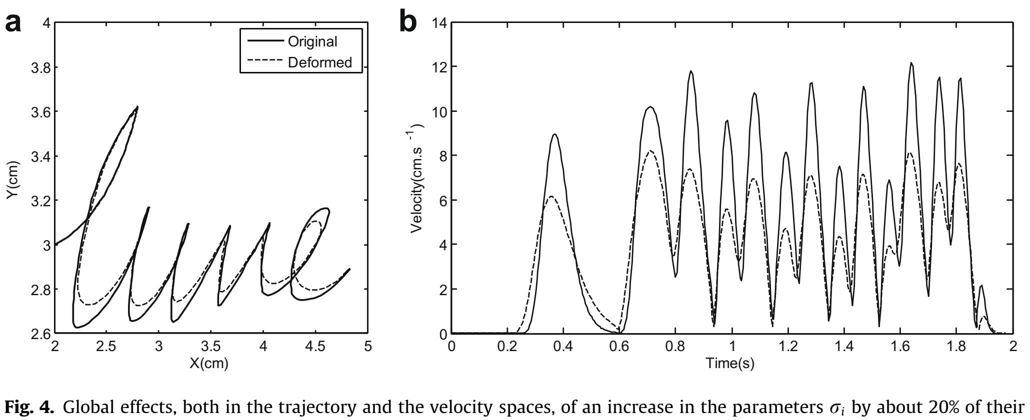

Fig. 4. Global effects, both in the trajectory and the velocity spaces, of an increase in the parameters σ_i by about 20% of their original values, which leads to a smoother pattern (the same effect is observed when increasing the parameters μ_i).

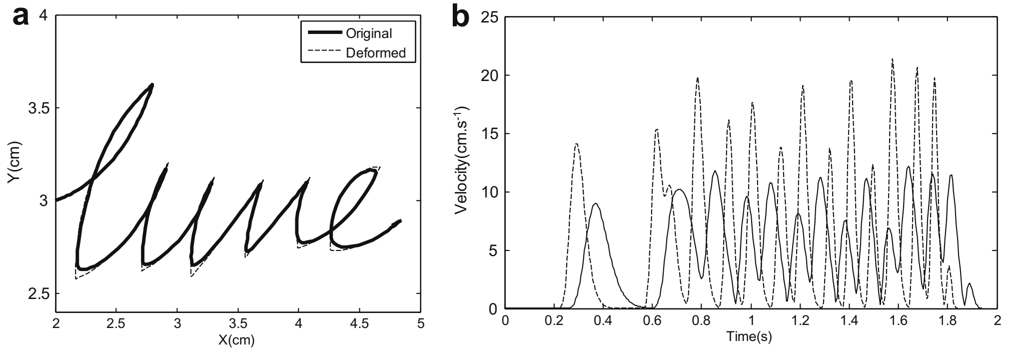

Fig. 5. Global effects, both in the trajectory and the velocity spaces, of a decrease in the parameters μ_i by about 10% of their original values, which leads to a sharper pattern (the same effect is observed when decreasing the parameters σ_i).

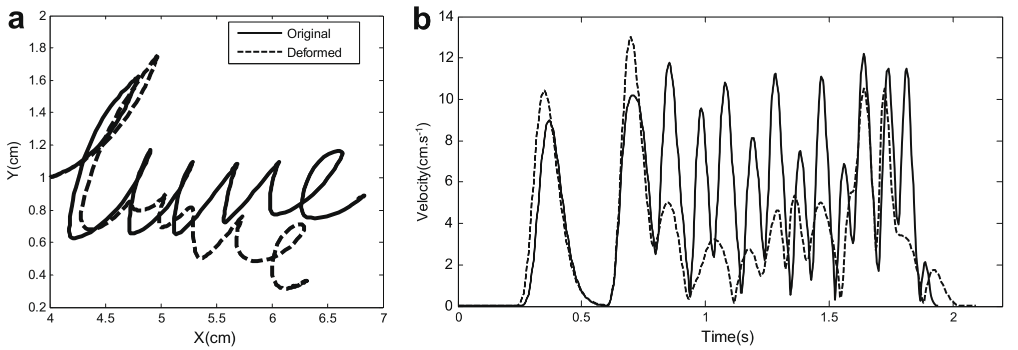

Fig. 6. Local variational effects of the Sigma-Lognormal parameters, both on (a) the trajectory and (b) the velocity profiles, when all the parameters are varied slightly.

5. Method

To act on the global variability of the handwriting scale (or to study the zoom effect), we can ask a writer to increase the global shape of a word by writing it in rectangles of different sizes, but with identical orientation. To act on the global variability of the direction, we can ask a writer to produce the same word inside rectangles of the same size, but with different orientations. This methodology can be considered as a potential way to control the magnitude and direction of a subject's handwriting externally.

5.1. Experimental procedure

Following the approval of the local ethics committee, experiments were performed on 6 right-handed subjects of both genders between the ages of 22 and 45 and in good health, i.e., without any declared history of neurological or physiological disease. They were asked to firmly grasp a stylus in their dominant hand, and, after hearing the audio signal "Bip", to write the word "lune" inside a rectangle on a digitizer (Wacom Intuos II, 22 $\times$ 32 cm, resolution 100 points per mm). The stylus pressure and the x–y trajectory were sampled at 200 Hz and the velocity profiles were numerically calculated using a derivative filter with Fc = 60 Hz and a Cheby II low-pass filter with Fc = 16 Hz, and Att = $-$81 dB. Fig. 7a illustrates the acquisition of a typical trial with the apparatus.

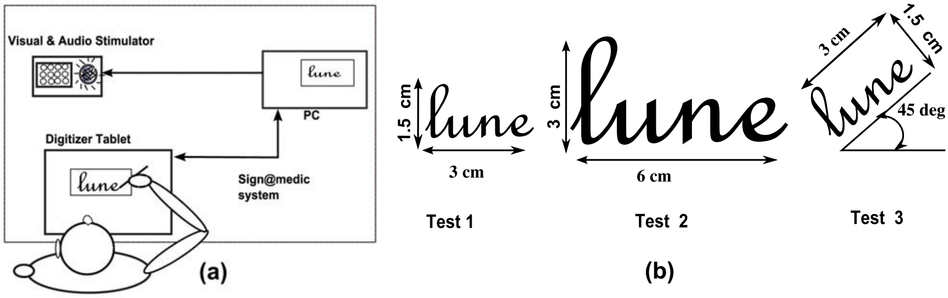

Fig. 7. (a) illustration of the apparatus and the experimental protocol used in the acquisition of handwriting. After receiving an audio stimulus, the writer writes a word in the specified rectangle, and the digitizer records the corresponding trajectory of the stylus; (b) illustration of the three tests used in this experiment.

The experimental protocol included a series of three tests repeated three times. In $Test_1$, considered as the baseline test, specimens of the word "lune" were acquired. The writers were asked to write horizontally, inside a 3 cm × 1.5 cm rectangle. In *Test*$_2$, a global zoom change was requested by asking writers to write the same word in a larger horizontal rectangle (6 cm × 3 cm) (we assumed that this window roughly corresponds to the writing surface of the most popular PDAs). In *Test*$_3$, global directional variability was required by asking the participants to write inside an oblique rectangle 3 cm × 1.5 cm at 35° relative to the horizontal (see Fig. 7b). Each test was repeated three times to build the specific fiducial pattern of the target word "lune" for each writer.

5.2. Preprocessing

The Sigma-Lognormal parameters that represent the best modeling of the handwriting pattern were estimated in two steps, using a parameter extraction system. The first step resulted in the estimation of an initial set of parameters, as obtained empirically after an interactive fitting of both the velocity profile and the *x*–*y* trajectory of the patterns with the Sigma-Lognormal model (Djioua, O'Reilly, & Plamondon, 2006). The second module started from these initial estimates and used the Levenberg–Marquardt nonlinear optimization algorithm to automatically converge toward an optimal solution, according to the mean square reconstruction errors (Levenberg, 1944; Marquardt, 1963). Fig. 8 depicts a typical excellent Sigma-Lognormal fit, both in the velocity and in the trajectory spaces, for a specimen made up of 14 strokes.

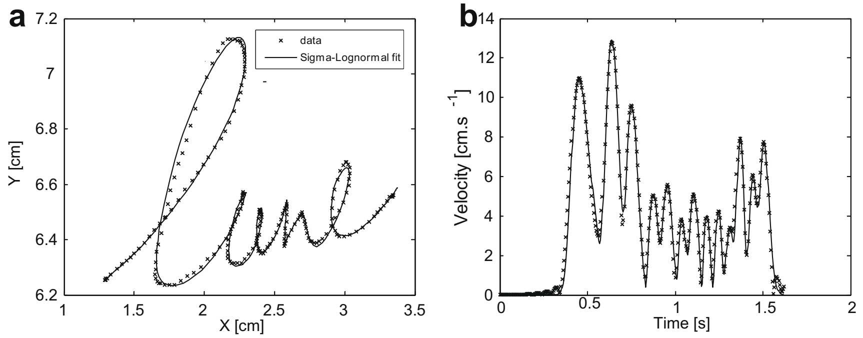

Fig. 8. Typical fitting by the Sigma-Lognormal model of the word "lune" depicted in (a) the trajectory and in (b) the velocity representation spaces. In this study, the glitches appearing below a first preponderant velocity peak are considered as local perturbations, and the first glitch appearing after the last preponderant velocity profile is considered as the antagonist response, corresponding to the 14*i*th stroke of a word pattern.

5.3. Normalization of Fiducial Patterns

For a given test, each subject carried out three trials from which a fiducial was built using the mean values of the extracted parameters. Fig. 9 depicts the fiducial patterns produced by the six writers over the three tests. In agreement with Eq. (4), the following normalization of the fiducial patterns was made to analyze the global variability:

$$D_i \rightarrow \frac{D_i}{D_{\max}}, \quad D_{\max} = \max\{D_i\} \qquad i = 1, \ldots, 14, \tag{5a}$$

$$\theta_{di} \rightarrow \theta_{di} - \theta_{d1}, \tag{5b}$$

$$\theta_{fi} \rightarrow \theta_{fi} - \theta_{d1}, \tag{5c}$$

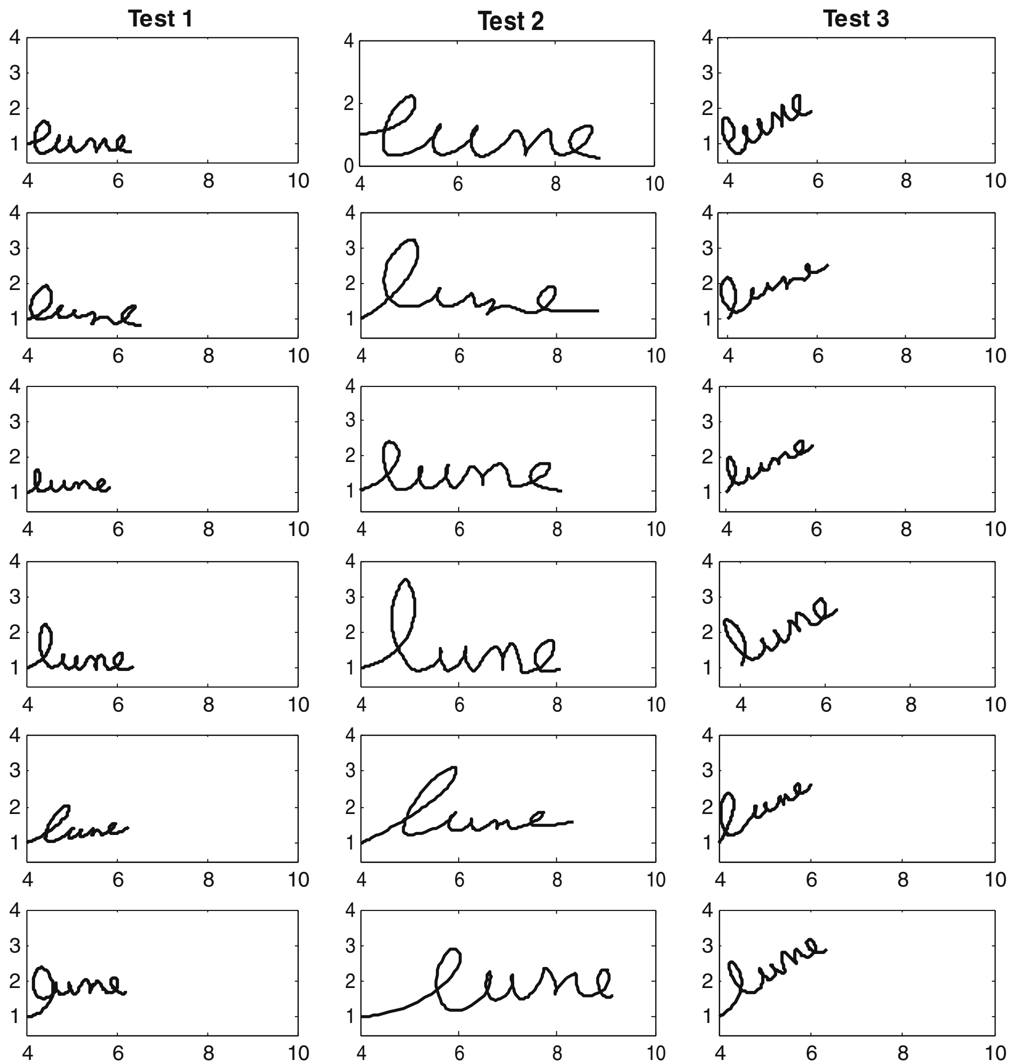

Fig. 9. List of fiducial patterns of the word "lune", as written by the six writers over the three tests.

$$t_{0i} \rightarrow t_{0i} - t_{01} = A(i-1), \tag{5d}$$

$$\mu_i \rightarrow \bar{\mu} = \frac{1}{14}\sum_{i=1}^{14}\mu_i, \tag{5e}$$

$$\sigma_i \rightarrow \bar{\sigma} = \frac{1}{14}\sum_{i=1}^{14}\sigma_i. \tag{5f}$$

In this normalization process, the maximum length of the strokes is made equal to 1 and $D_{\max} = \max\{D_i\}_{i=1,\dots,14}$ is then considered as the zoom factor. Furthermore, the start angle θ_{d1} of the first stroke is considered as the initial direction. Other normalizations were made to characterize each fiducial with single values representing its 14 constituent strokes. The time-based peripheral parameters are described by their mean values $(\bar{\mu}, \bar{\sigma})$, and the timing of the motor plan is represented by the slope A of a $\{t_{0i}\}_{i=1,\dots,14}$ sequence.

6. Analysis of the results

An ANOVA performed on the feature vector components of the fiducial patterns revealed that the mean values of $D_{\max}$, θ_{d1} and θ_{f1} were significantly different in the three tests, with a maximum p-level of .003 (see Table 1).

In Fig. 10a, a significant variation of $D_{\max}$ is reported: the mean values increased by a factor of 2.08 (see Table 2: row 1, columns 1 and 2) when the writers were asked to enlarge their handwriting by a factor of 2 (from $Test_1$ to $Test_2$). Through this analysis, one can also deduce that the pattern size remained approximately the same when comparing $Test_3$ to $Test_1$ (see Table 2: row 1, columns 1 and 3).

Table 1
ANOVA analysis: univariate results for each dependent variable.

Effect	A		$D_{\max}$		θ_{d1}		θ_{f1}		μ_{mean}		σ_{mean}		DOF
	F-val.	p-val.	F-val.	p-val.	F-val.	p-val.	F-val.	p-val.	F-val.	p-val.	F-val.	p-val.	
Test	0.282	.757	24.864	.000	17.184	.000	8.839	.003	0.370	.696	0.222	.803	2

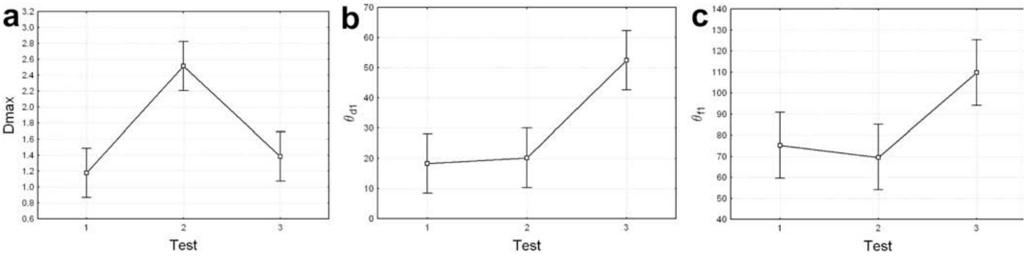

Fig. 10. Summary of the variations with tests for the feature vector components $D_{\max}$, θ_{d1}, θ_{f1}.

Table 2
Mean values of the amplitude and directional parameters, obtained from each test.

	Test 1	Test 2	Test 3
$D_{\max}$ (cm)	1.2	2.5	1.4
θ_{d1} (°)	18	20	52
θ_{f1} (°)	75	70	110

To sum up, the vector $(A, \max\{D_i\}, \theta_{d1}, \theta_{f1}, \bar{\mu}, \bar{\sigma})$ is considered as the feature vector of the fiducial patterns from which the two predictions under study can be analyzed.

In the case of directional variability, the results summarized in Figs. 10b and c show that the differences were the most striking for θ_{d1} and θ_{f1}, where the mean values increased by 34° and 35°, respectively, when the writers were asked to rotate their handwriting by an angle of 35° (from $Test_1$ to $Test_3$) (see Table 2: rows 2 and 3, columns 1 and 3). As highlighted in Table 1, the uncontrolled variations of the other components of the feature vector were non-significant, with p-values >.05.

This is consistent with the predictions of the Kinematic Theory, as simulated in Section 4 by the Sigma-Lognormal model.

The ANOVA analysis corroborates the fact that there are significant global variations in the amplitude and direction parameters, even in the presence of local variations in the Sigma-Lognormal parameters, with factor values close to those required by the experimental protocol. As emphasized in Section 4, when these global variations are assumed to occur, either from homothetic changes in D_i or from a systematic offset in the θ_{di}, θ_{fi}, there should be no significant effect on the time scale of the corresponding complex movements, and, furthermore, the amplitude of the velocity profile should not be affected by the rotational effects. These latter predictions are globally confirmed in Fig. 11, where the proportionality of the movement times (MT) is analyzed. Indeed, the proportional regression curves describing the movement times MT_3 of the $Test_3$ patterns and MT_2 of the $Test_2$ patterns versus the movement time MT_1 of the referential $Test_1$ pattern have slopes close to 1, at 1.09 and 0.94 respectively, which reflects the fact that the movement duration remained globally constant under scale changes or rotations. This result is still valid under the assumption that the number of strokes involved in the production of a pattern is constant over scales and because writers have written a word with smaller letters (under 3 cm). Indeed, when the number of strokes and the time plan $\{t_{0i}\}$ are constant, this result can be explained by the isochrony principle, where, for writing movements, the duration tends to remain constant across changes in trajectory length. It has been shown that, in the case of the production of large letters, the movement time will increase when the scale increases (Teulings & Schomaker, 1993; van Doorn & Keuss, 1993; Wright, 1993). This prediction mainly results from the speed/accuracy trade-offs, where writers must reduce their movement speed by increasing the movement time to produce large and accurate strokes while at the same time preserving a word's shape (Plamondon & Alimi, 1997). The Kinematic Theory proposes an analytical expression which describes a relationship between the duration MT of a rapid movement producing a stroke and the peripheral parameters (μ, σ) (Djioua & Plamondon, 2008) to process these cases. However, these equations might not be necessary in the PDA context, where the writing surface already limits scale changes.

From a local perspective, each fiducial pattern has its own local variability, and this is reflected in the superimposition of different velocity profiles, as depicted in Figs. 12a and b, where deformations

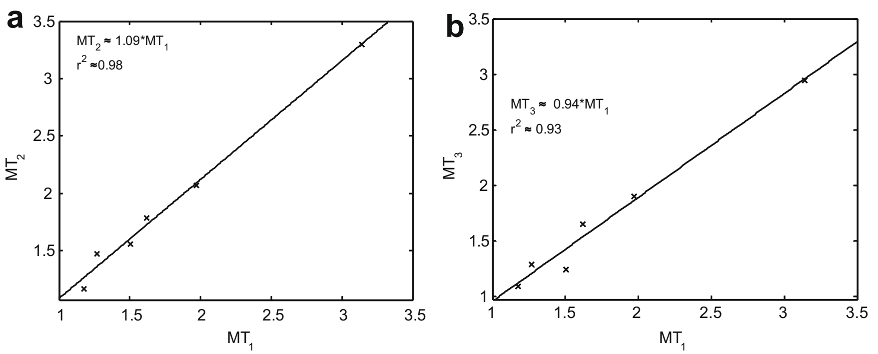

Fig. 11. Regression results of the fiducial movement times: (a) MT_2 versus MT_1; and (b) MT_3 versus MT_1, which show the preservation of the time scale when the six subjects scaled and rotated the word pattern respectively.

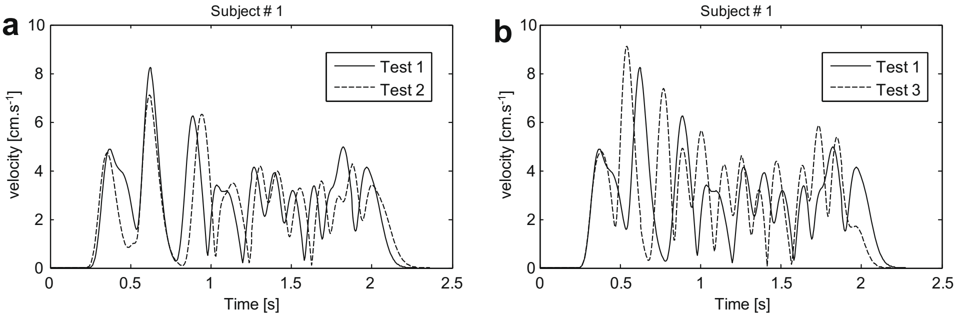

Fig. 12. Typical superposition of velocity profiles corresponding to the fiducial patterns constructed from the (a) $Test_1$ and $Test_2$, and (b) $Test_1$ and $Test_3$ data of a writer #1, after normalizing the scale (using Eq. (5a)). The time scale is more or less locally preserved, depending on the residual local variability of the parameters.

caused by the unavoidable local variations in the parameters is apparent. In other words, instead of obtaining superposition similar to the ideal prediction illustrated in Figs. 2c and 3c, the residual local variability has led to distortions that are similar to those shown in Fig. 6b.

7. Conclusion

In this study, specific predictions of the Kinematic Theory regarding some possible causes of handwriting variability have been formalized and tested, both with computer simulations and experimental investigation. The pattern variability of a word has been analyzed through the fluctuations of its Sigma-Lognormal parameters, and global and local phenomena have been identified and modeled. The resulting simulations have provided information about the individual effect of each parameter on the predicted variability. From a global variability perspective, it has been shown that, if the same preprogrammed action plan is used, the motor control system should produce a word under scale changes and directions without affecting the synchronicity of the task execution process, as reflected by the stability of the velocity profile. In contrast, variations in the peripheral parameters that describe the time-based behavior of the neuromuscular system would act on the smoothness and sharpness of the handwriting patterns, directly de-synchronizing the velocity patterns.

An experiment has been conducted to investigate two of these predictions: one associated with a scale change and the other with a rotation. The results presented through an ANOVA analysis corroborate both sets of predictions. It was observed that the subjects could voluntarily produce a scale change without significantly affecting the timing of their velocity profiles, that is, only the amplitude patterns were directly re-scalable by preserving the number of strokes. Similarly, the participants were able to change the global orientation of their trajectories without notably affecting their velocity profiles. These results are in accordance with the basic representation proposed by the Kinematic Theory: a set of commands that control, at a high level of organization, both the amplitude and the direction of a movement under a specific timing sequence. Regarding the other theoretical predictions made in this paper, such as those dealing with the variability of the preprogrammed time plan, as represented by a sequence of $\{t_{0i}\}$, and those concerning the variability of a neuromuscular system, as described by its peripheral parameters (μ_i, σ_i), this knowledge can be of great value in the design of an incrementally adaptive recognizer. However, it might be quite difficult to design psychophysical experiments which will easily provide some external control over these parameters, particularly in the context of writing small letters (< 3 cm) in applications dealing with the generation of word databases, such as those recorded with PDAs and smartphones. Although there is still a long way to go, we expect that our work here will provide interesting opportunities for both motor control studies dealing with handwriting generation and projects aimed at the automatic generation of huge databases of letters and words for the development, training, and testing of on-line handwriting classifiers and recognizers (Djioua & Plamondon, in press).

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