Acta Psychologica 54 (1983) 99–114
North-Holland

ANALYSIS OF HANDWRITING MOVEMENTS

E.H. DOOIJES *

University of Amsterdam, The Netherlands

Accepted May 1983

In the early sixties a theory for the generation of the fast handwriting movements was proposed by Denier van der Gon. This theory asserts that the physiological mechanism involved can be described by a pair of independent open-loop systems of low order, both excited by bang-bang type signals. These input signals represent the neural signals from the brain, where motor programs for the production of primitive symbols are thought to be stored in some simply coded form. In this paper we describe our efforts to reconstruct the motor programs from observed time signals, representing the pen movements during the performance of natural writing tasks by normal, experienced subjects.

1. Introduction

In this paper we discuss a system-analytical approach to the description of the dynamics of fast, cursive handwriting. Thus, we interpret the observed writing movements as responses of a model of the physiological mechanism to a certain class of input signals; our model is based upon the work of Denier van der Gon et al. (1962, 1965).

In the present problem, input signals cannot be measured directly. On the other hand, rather strong assumptions can be made about their general shape. Now we seek to answer the question whether these assumptions are sufficient to permit

(a) the determination of the unknown model parameters, and
(b) the reconstruction of the driving signal (the "motor program").

* The author is indebted to the Laboratory of Medical Physics for giving him the opportunity to do this work as a guest researcher. Also, he wishes to thank the reviewers for their helpful criticism.
Author's address: Computer Science Dept., University of Amsterdam, Plantage Muidergracht 6, 1018 TV Amsterdam, The Netherlands.

We investigate techniques for the constructive solution of problems (a) and (b). These techniques are tested on a limited number of samples of natural, cursive writing, produced by reportedly normal, right-handed subjects. The paper is organized as follows. Section 2 covers a concise description of Denier van der Gon's model of the handwriting mechanism. In section 3 we give some details about the recording and preprocessing of the writing movements.

The recorded pen movements have to be resolved into components which can be considered to represent the contributions of the individual muscular actuators involved in the writing mechanism. We have devoted section 4 to a somewhat expanded description of the techniques developed for the solution of this problem.

Section 5 deals with the problem of estimating both the structure of the motor program and the parameters of the model system. In section 6 we discuss the role played by the "writing pressure" in the guiding of the pen. In these sections emphasis is on the conclusions, rather than on the technical background. Finally, in section 7 we present an overview of our results.

A more detailed exposition of the topics of this paper will be given in the author's doctoral thesis (Dooijes 1983). Some of the basic ideas presented in section 5 have been published earlier as a conference paper (Dooijes 1980).

2. Basic assumptions

The model which constitutes the basis of our investigations has been developed by Denier van der Gon and co-workers (1962, 1975). They argued that the physiological mechanism which is responsible for the fast handwriting movements can be described by a pair of open loop dynamical systems of low order, both excited by two-level bang-bang signals. These systems represent the force generators – pairs of antagonistic muscles, making the pen move with two degrees of freedom – and the moving parts, contributing inertial, frictional and elastic terms to the systems. The force exerted by an excited muscle is thought to be at a constant level in the course of writing a single word (apart from switching transients), this level being predetermined in accordance with the desired overall size of the word; the information determining the shape of the individual script primitives is contained in the timing of the switching instants. The slow left-to-right motion superimposed on the faster writing movements is thought to have a different, independent origin.

The feasibility of this theory was demonstrated by Denier van der Gon et al. (1962) by means of a physical simulation model. The motor programs were obtained by trial and error, the model systems being fixed and approximately ballistic (i.e., with predominating inertial terms). Koster and Vredenbregt (1971), working along the same lines, were able to show that, in conformity with the theory, the timing of the motor programs is highly specific for the symbols being simulated, regardless of their size.

In a sense, the present work can be viewed as an attempt to automatize this analysis-by-synthesis procedure, with the important difference that the temporal aspects of the real writing samples are explicitly taken into consideration. Indeed, it can be shown that the simulation problem is trivial (i.e., it is solvable regardless of the adequacy of the underlying model) without this added constraint. Moreover, this approach leads us to the conclusion that the original model has to be revised in some respects to provide an adequate description of the data.

3. Recording of the pen movements

Subjects are requested to write on a 10×10 cm^2 paper surface without further constraints (such as a prescribed baseline), using a specially prepared ball-point pen. The pen position is sensed by an optical-electronic device (Loehnberg 1963), which for our purpose can be considered to be free from bandwidth and resolution limitations. A major feature is its ability to follow the stylus even if it is raised from the writing surface up to about 5 mm. On top of the pen a miniature force transducer is mounted, with a range of 0–5 N and proof against considerable overload. This transducer measures the axial component of the "writing pressure".

The pen position, resolved into Cartesian X and Y coordinates, and the writing pressure (Z) signal are represented by analog voltages. These are sampled at 100/s with 11 bits resolution after passing a 30 Hz 4-pole low-pass filter. Before further processing, the X and Y signals pass a non-recursive digital filter with 13.6 Hz cut-off frequency. This filter spends 20 data points at both signal ends.

Here we comment on the common misconception that the passband of the preprocessing filter should be chosen so as to pass a significant portion of the frequency band associated with the bang-bang signal (motor program) we attempt to reconstruct. This would require a filter bandwidth at least 5 times as large as we actually use, and a correspondingly higher sampling rate.

However, on inspecting the power spectra of the recorded displacement signals it is readily observed that for the major part of the Nyquist frequency band 0–50 Hz any eventual signal component is well below the noise level; hence by extending the filter bandwidth no real information is gained in the first place. The apparent paradox that from the filtered signal a motor program is computed with substantially higher bandwidth is resolved if it is realized that information is added by assuming a specific (bang-bang) shape for the motor program.

4. Decomposition of the displacement signals

In our model, defined in section 2, three degrees of freedom are involved in the production of handwriting. One of these is associated with the relatively slow left-to-right motion, the remaining two with the fast components.

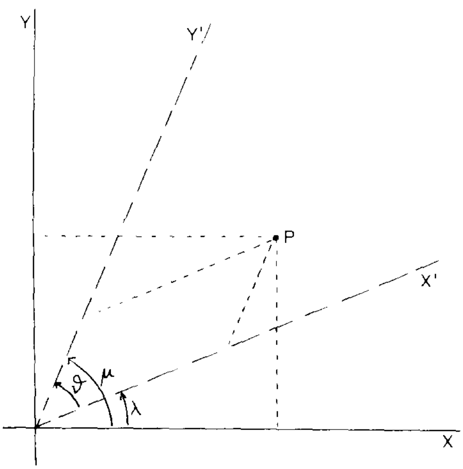


Fig. 1. Reference frames: XY Cartesian coordinate system, X parallel to writing base line; X'Y' principal coordinate axes.

Let us call *principal directions* of motion those emerging if either of the actuators, associated with these last degrees of freedom is excited separately. Still according to the model, these actuators are antagonistic muscle pairs, representable by differential systems, independently excited by bang-bang signals.

The recorded movements are projections of the principal movements onto orthogonal coordinate axes, fixed to the recording apparatus, and as such linear combinations of the latter. It will generally not be possible – or at least not physically meaningful – to interpret these combinations likewise as the outputs of systems excited by bang-bang signals. Therefore, we have to perform a transformation of the recorded signals, defined in a Cartesian XY coordinate system, to a new, generally oblique system X′Y′, moving left-to-right with respect to the first one (fig. 1).

The X direction of the Cartesian coordinate system is taken to coincide with the virtual writing base line, the estimation of which is described in section 4.2. As there is no prescribed baseline indicated on the writing sheet, a rotation operation is generally needed to realize this situation. In the remainder of this section we consider a number of approaches for identifying and separating the signals associated with the left-to-right motion (often called "trend" in the sequel) and both of the principal movements.

4.1. Kinematic considerations

By inspection of the writing gestures it is possible to identify the linkages involved with the rapid writing movements. The observations reported here hold for all of our subjects and presumably for most normal, right-handed writers.

One of the linkages concerned is associated with a rotational motion of the hand in its entirety with respect to the fore-arm, about a dorsal axis located in the wrist joint. From X-ray studies by Andrews and Youm (1979) the normal wrist joint is known to behave as a tight joint within a range of plus and minus 20° from the stretched position. Correspondingly, if the fore-arm remains stationary, the pen tip describes a planar circular arc which has, for a typical subject, a radius R of approximately 15 cm. As the deviations, measured at the pen tip, are of order 0.5 cm, the pen motion can be considered to be rectilinear, its direction being fixed for a given position of the hand. This direction is to be identified with the X′-axis.

Motion of the pen tip in the second principal direction, Y′, is caused by moving up and down the upper two phalanges of the forefinger. It is not difficult to visualize both the X′ and Y′ axes by in turn putting into motion the corresponding mechanisms. This simple experiment has been repeated quite a few times over an extended period (a year); it turns out that the angle ϑ between both axes (see fig. 1) is reproducible with remarkable accuracy. This observation also holds for less common attitudes of the hand, for instance that employed on sketching. A convenient parameter to express the attitude is the distance R of the pen tip to the center of rotation.

For the average cursive writing attitude, the angle ϑ varies from 45° to 90° among our subjects. The often quoted view that the principal directions are about perpendicular in general is not supported by these observations. While the relationship between R and ϑ is quite reproducible, it appears that R (and hence ϑ) cannot be regarded as constant during the writing of a complete sentence. For most subjects it seems reasonable to assume, however, that R and ϑ are constant within a single word – not excluding a possible shifting of the center of rotation.

4.2. *Identification of the trend component*

In general, the slow left-to-right motion of the hand is controlled by visual and/or kinesthetic feedback. The description of this type of motion in terms of a process-model with identifiable parameters presents serious difficulties. Therefore we choose to isolate the trend component on the basis of its assumed uniformity. This assumption is a corollary from the idea that the trend mechanism operates independently of the mechanism responsible for the fast, letter-forming movements. A uniform "time-base" is then the obvious way to achieve an even appearance of the script pattern, which is indeed observed with normal, experienced writers. In what follows we will provide further evidence for this assumption (section 4.3). Thus, the left-to-right motion is isolated by fitting a linear function of time to the pen displacement data. The resulting virtual base line is identified with the X-axis of the Cartesian XY system (fig. 1). The result of this procedure is shown in fig. 2 for the test word *Amsterdam*, that will serve as an example throughout this paper.

We point out that this simple procedure is not always successful. For instance, in short words with a skew velocity distribution (like *momom*)

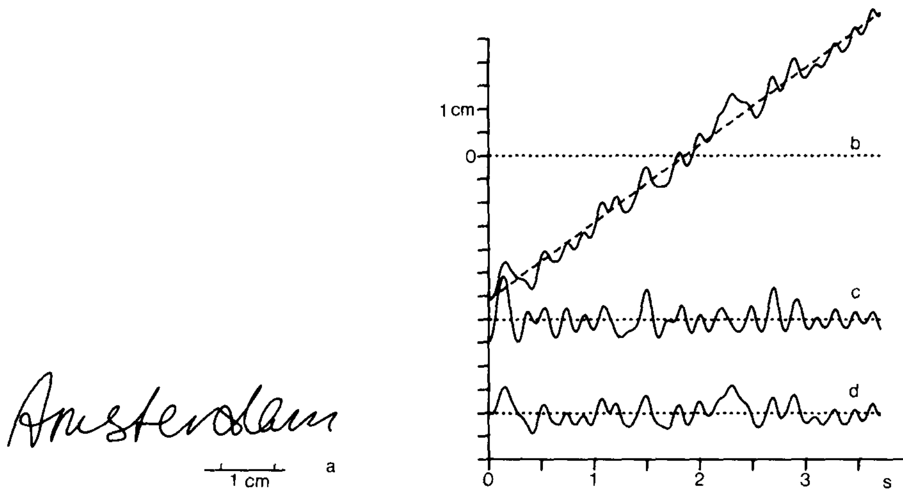


Fig. 2. (a) Original script pattern. (b) X-component; dashed: uniform trend. (c) Y-component. (d) X-component, trend removed.

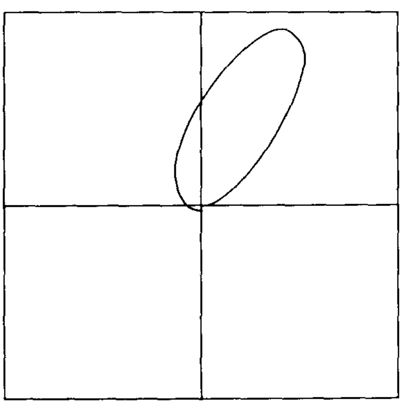
fitting a uniform trend leads to underestimation of the overall trend velocity. However, alternative methods for trend isolation are not suitable either. Low-pass filtering, for instance, has the essential problem that the transient decay time of the required filter is large (typically 20–30 percent) as related to the total signal duration.

4.3. Inferences from the writing angles distribution

The procedure discussed in this section is based upon the idea that the orientations of the principal directions in some way manifest themselves in the global aspect of the script pattern. More specifically, we discuss how information about the angles λ, μ (fig. 1) can be extracted from the distribution of the directions φ of tangents, measured at points uniformly spaced along the writing curve. Such a distribution is to be interpreted as an attribute of the particular sample word for which it is derived; hence it should not be regarded as an estimate of a hypothetical writing angles distribution characteristic for the subject in question. The distribution, represented by a continuous probability density function (pdf), is estimated from a histogram (bin width 15°) of φ-values, measured at the sampling points. However, the writing line is sampled at points equidistant in *time*. Hence, evaluating φ on a mesh uniform in *distance* along the writing line would require interpolation between the sampling points. To avoid this, we compute φ in each of the available points, assigning it a weight proportional to the absolute velocity at the corresponding sampling instant. The (estimated) continuous pdf is obtained by fitting a periodic spline of order 4 to the histogram data. As the pdf is somewhat difficult to interpret, we introduce a procedure which we call Lissajous Transformation (LT). Starting from the writing angle pdf, this procedure constructs a simple, non-intersecting planar curve with a distribution of tangent directions identical to that of the original script pattern. This transformation has the following rather self-evident invariance property: all -not necessarily congruent- patterns with the same pdf map onto a single transform pattern, irrespective of their absolute sizes.

The Lissajous transforms for simple test words like *eeeee* or *lllll* are easy to interpret, as the x' and y' components of the displacement signals are approximately sinusoidal. (Denier van der Gon's theory asserts that these components consist of sequences of contiguous parabolic segments; however, such a sequence can be approximated by a sine wave with an average error less than 3 percent.)

If both the x' and y' signals were purely sinusoidal, without a trend component, then the transform pattern would be a simple Lissajous ellips; and this is what we actually find for words of the type indicated



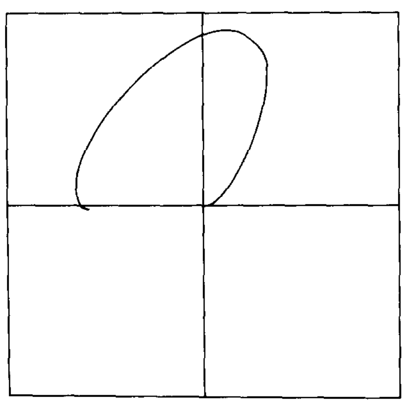


Fig. 3 (*left*). Lissajous transform pattern; trend removed.

Fig. 4 (*right*). Lissajous transform pattern; trend not removed.

above, presupposed that prior to computation of the pdf the left-to-right trend has been removed using the technique discussed in the previous section. Similar results are obtained for the test word *elelele*; this is to be expected in view of the invariance property of the Lissajous transformation.

The geometrical properties of the LT ellips – the angle of inclination of the major axis and the lengths ratio of the major and minor axes – are readily measured with a ruler. It is, however, not possible to infer from these quantities the angles λ, μ (fig. 1) defining the X' and Y' coordinate axes, unless additional information is present. Hence we exploit our knowledge of the included angle ϑ (section 4.1) and, moreover, assume that the x' and y' signals exhibit a sine-cosine relationship. This assumption formalizes the idea that a characteristic slant in the script pattern emerges if instants of maximal velocity in one (principal) direction tend to coincide with instants of zero velocity in the other direction. Notice that the qualification "sine-cosine relationship" should be loosely interpreted, as the signals have varying amplitude and – as mentioned before – may well consist of parabolic segments instead of sinusoids. With this additional information it is possible to compute λ, μ at least for the class of test words considered above.

Applying this technique to the more complicated test word "Amsterdam" (fig. 2) we find the remarkably regular LT pattern shown in fig. 3. Fig. 4 is obtained for the same test word without trend removal: notice the lack of closure of this LT pattern.

The geometrical properties of fig. 3, interpreted as an ellips, are not difficult to estimate. Taking the value of ϑ found for the subject's average writing attitude as described in section 4.1, we find $\lambda = 12.6°$, $\mu = 68.8°$, both within 1° of the principal directions found from the test word *elelel* written by the same subject. Similar results are found for the word *Amsterdam* written by other subjects in various sizes. (Of course, other values for λ, μ and ϑ hold in these cases.) Less ellips-like are the patterns for *momom*, which can be explained from the over-representation of certain features in this word (see also section 4.2). However, in most cases studied the Lissajous transform – if computed after trend removal – is a nearly closed curve. We hold this as an indication that the left-to-right trend has been correctly identified as uniform and rectilinear. One could imagine that these closed curves come about by a continued looping of the pen tip, the sense of rotation being reversed every now and then.

4.4. A self-consistent procedure for the determination of the principal directions

A major assumption underlying the Lissajous transform method for the determination of the principal directions is that instants of maximal velocity in one (principal) direction tend to coincide with instants of zero velocity in the other direction. For quasi-sinusoidal signals such as those appearing in the test word *elelele*, this "reciprocal motion" (RM) condition is fulfilled if there exists a "sine-cosine" relationship between the X′ and Y′ components. That the Lissajous method gives consistent results also for more complicated testwords suggests that the RM condition holds in this case too.

This condition is strictly fulfilled for the class of so-called Hilbert transform pairs. (A Hilbert transformer is a non-causal filter introducing a 90° phase shift while having a flat amplitude response over its pass band.) Although it seems unlikely that actual writing signals exhibit a simple linear relationship, as do Hilbert transform pairs, and a *strict* RM relation is neither to be expected, the Hilbert transform model suggests a simple procedure for the determination of the principal directions. This procedure amounts to searching for a pair of principal direction λ, μ such that the cross-correlation function (ccf) of the corresponding x' and y' signals is anti-symmetrical; it does not depend on external information regarding the value of ϑ, the angle included by the principal directions.

For most test words the sense of rotation reverses several times in the course of writing. Each reversal implies a 180° phase reversal of one of the component signals at a time, which is not in agreement with the idea that both signals constitute a Hilbert transform pair. Hence we would expect at best a piecewise Hilbert relationship, the role of $x'(t)$ and $y'(t)$ as input and output of a virtual Hilbert transformer alternating at each turning point. A piecewise Hilbert relationship, in this sense, still has the effect of producing an antisymmetrical ccf, which can be regarded as the cumulation of a number of ccf's corresponding to short Hilbert-pair segments. In view of this, strict antisymmetry of the bulk ccf will be confined to a relatively small neighbourhood of lag zero. (In fact, other factors such as inadequacy of the inherently linear Hilbert transform model also contribute to this phenomenon.)

On applying this procedure to our collection of test words, a unique λ, μ pair is found for all but the simplest test words. That simple words like *elelel* do not contain sufficient information for the determination of the principal directions should indeed be expected, as for (quasi)sinusoidal signal pairs an anti-symmetrical ccf is found for any of the infinitely many λ, μ pairs where the x' and y' signals have a sine-cosine relationship (or: zero correlation).

A result which confirms the validity of this approach is that for suitable test words – like *Amsterdam* – the principal directions λ, μ found this way are – within natural limits – in accordance with the values derived from the Lissajous transform procedure. This means in particular that in each case $\vartheta = \mu - \lambda$ agrees with the value obtained independently from the experiment described in section 4.1.

5. Reconstruction of the motor program

The problem we are dealing with in this section can be formulated as follows:

> Construct a bang-bang input signal u and a set P of parameters and initial conditions for the black box S (fig. 5), such that the output signal x approximates the observed signal X according to some suitable error norm. Here X may represent either of the principal displacement signals. The sign reversals of u divide the time interval where X is defined in N contiguous segments, on each of which u takes on a constant value with alternating sign; N is not known beforehand.

As it turns out, any algorithm eventually leading to a provable solution is prohibitively expensive in terms of computation time. However, by applying the technique of *dynamic programming* (Bellman et al. 1965) a possibly sub-optimal solution can be constructed in a relatively short time, presupposed that the range of admissible values of the parameters

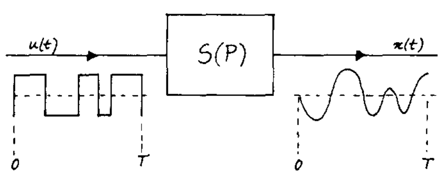


Fig. 5. Model system.

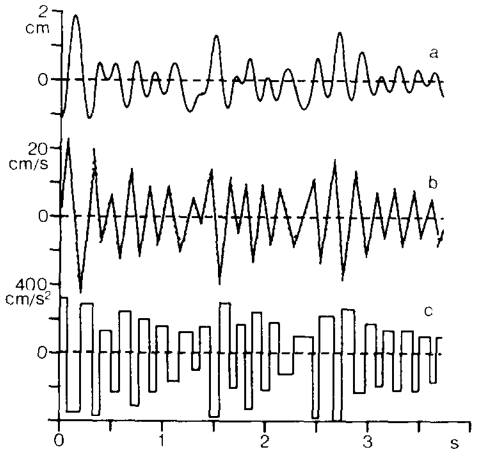


Fig. 6. Reconstruction of Y′ component: (a) displacement; (b) velocity, dotted: original signal; (c) acceleration.

P is confined as far as possible, which is feasible only if we have some a priori knowledge of the expected results. (This is a common problem in the identification of physiological systems; see Bekey 1973.) For the same reason it is important to restrict the number of free parameters in the model S. In view of these difficulties we adopt the particularly simple model

$$S\colon d^2x/dt^2 = u(t)$$

rather than one suitable to accommodate all of the modalities mentioned in section 2.

S is represented here by the equation of motion of an inertial mass m acted upon by a force $m.u$. Considering the results of Denier van der Gon, we expect that this purely ballistic model can serve at least as a first approximation.

Our experimental results indicate that the observed signals can be described satisfactory in terms of this model (see fig. 6). However, to

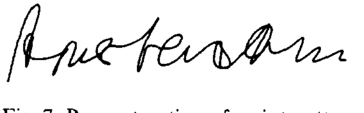


Fig. 7. Reconstruction of script pattern from motorprogram.

attain this result the original postulate regarding the strictly two-level bang-bang character of the input signal u has to be relaxed. Further analysis indicates that neither shifting of the computed switching instants, nor enhancement of S by terms of higher order (bringing into acount the muscle activation and deactivation time) is likely to save the two-level input model.

Approximate reconstruction (fig. 7) of the original script pattern from the computed motor programs is possible if the ballistic model S is extended with frictional and elastic terms, estimated with the aid of a search procedure:

$$S'\colon m.d^2x/dt^2 + r.dx/dt + k.x = m.u(t)$$

The damping term found for this new model S' is always far more important than the elastic term. Typical values for the parameters of S' are

$$m = 0.05\ kg;\ r = 0.6\ Ns/m;\ k = 0.002\ N/m.$$

It can be shown that the characteristic times associated with these additional terms are large as compared to the time scale of the motor program. Therefore the apparent ballistic behavior of the physiological system is not in contradiction with the presence of non-inertial forces.

6. Writing pressure

The writing pressure – or properly speaking, the component of the force exerted on the writing surface, measured in the direction of the pen's axis – shows a markedly regular and reproducible pattern. As the pressure variations can affect the dynamics in the XY plane only through the accompanying fluctuations in friction, a laboratory experiment has been performed to investigate how both quantities are related for the ball-point pen and paper used in our recording apparatus. It is found that the frictional forces are mainly of the Coulomb type, and that they are, on the average, of the order of magnitude of the inertial forces – a somewhat surprising result in view of the merits of the ballistic model.

Applying the describing-function method it is possible to estimate how Coulomb friction – a non-linear phenomenon – manifests itself in the linear model S' (section 5). It appears that the amount of friction encountered in our laboratory experiment is in quantitative agreement with the value of the damping coefficient computed for model S'.

7. Results and discussion

The main conclusions from the previous sections can be summarized as follows.

(1) The left-to-right motion (trend) is adequately described as uniform and rectilinear, as assessed from the observation that the residuals (representing the fast writing movements) project onto a coordinate system moving with the trend velocity as stationary closed loops. More involved trend models give rise to identifiability problems, due to the short signal duration.

(2) Application of the reciprocal-motion criterion for the identification of the principal directions gives consistent results for different test words written by a single subject; the directions found agree with independent estimates obtained from direct inspection of the hand-pen system. The fact that the principal directions are, for some subjects, far from perpendicular emphasizes the necessity to perform an axes transformation prior to further analysis of the recorded signals.

(3) Very simple test words (like *elelel*) or more complicated words in which certain features are over-represented (like *momom*) are not very suitable for analysis by the methods presented in this paper. Fairly complex – yet familiar – words like *Amsterdam* seem to fit best the purpose.

(4) Our analysis suggests that the model by Denier van der Gon is not entirely correct. So far as it asserts that the process behaves as a nearly ballistic system, driven by a *piecewise* constant alternating force, it is in agreement with our observations, which furthermore suggest that switching transients are not very important relative to the characteristic time scale of the motor program. However where Denier van der Gon's model assumes that the force levels are also constant *globally*, i.e. over the duration of an entire sample word, it is in contradiction with our findings. The apparent ballistic nature

of the writing process by no means implies that friction phenomena are negligible. However elasticity seems to play an insignificant role.

(5) The reconstruction of motor programs in the sense of this paper is a problem at the boundary of tractability. This boundary is likely to be crossed if more involved models for the writing process are introduced.

An example of further use of the motor programs computed by the procedures described in this paper is the following: Comparing the Y'-component motor program timing patterns for the test word "Amsterdam", written by a single subject with various sizes and attitudes (for instance, backwards slanted), a satisfactory similarity is found. Instead of comparing the time durations T for corresponding segments, the ballistic nature of the writing process suggests comparing the quantities T^2u, where u is the amplitude, as before. Invariance of the quantity T^2u implies that the physiological mechanism is able to trade force for time duration. Indeed it is oberved that, if assessed on the basis of this quantity, the correspondence between motor programs is much better than judged from the timing patterns only.

The internal consistency of the results reported above gives us some confidence as to their physiological relevance. However, a valuable extension to the present work would be a study of the relationship between our motor programs and (for instance) EMG data, recorded simultaneously with the writing movements.

We have applied our procedures to a relatively small number (about 20) of actual writing samples. However, as our concern has been mainly with methodological an computational aspects, we do not consider this as a real deficiency, relying that the samples chosen are sufficiently representative for normal handwriting. Indeed, it is difficult to assess how the plausibility of results like ours – which cannot be expressed in statistical terms – depends on the number of cases studied.

References

Andrews, J.G. and Y. Youm, 1979. A biomechanical investigation of wrist kinematics. Journal of Biomechanics 12, 83–93.

Bekey, G.A., 1973. 'Parameter estimation in biological systems: a survey'. In: P. Eykhoff (ed.), Identification and system parameter estimation. Amsterdam: North-Holland.

Bellman, R., B. Gluss and R. Roth, 1965. Segmental differential approximation and the 'black box' problem. Journal of Mathematical Analysis and Applications 12, 91–104.

Denier van der Gon, J.J., J.Ph. Thuring and J. Strackee, 1962. A handwriting simulator. Physics in Medicine and Biology 6, 407–413.

Denier van der Gon, J.J. and J.Ph. Thuring, 1965. The guiding of human handwriting movements. Kybernetik 2, 145–148.

Dooijes, E.H., 1980. 'A description of handwriting dynamics'. In: L. Dekker (ed.), Simulation of systems '79. Amsterdam: North-Holland.

Dooijes, E.H., 1983. Analysis of handwriting movement. Doctoral thesis, University of Amsterdam. (To appear).

Koster, W.G. and J. Vredenbregt, 1971. 'Analysis and synthesis of handwriting'. In: Biomechanics International Seminar II. Basel/New York: Karger.

Loehnberg, P., 1963. Master thesis, Delft Technical University. (In Dutch).