Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Human Movement Science 25 (2006) 586–607
HUMAN MOVEMENT SCIENCE
www.elsevier.com/locate/humov

A multi-level representation paradigm for handwriting stroke generation

Réjean Plamondon *, Moussa Djioua

Département de Génie Électrique, Laboratoire Scribens, École Polytechnique de Montréal, P.O. Box 6079, Station Centre-Ville, Montréal, Canada H3C 3A7

Available online 4 October 2006

Abstract

The study of rapid strokes is a direct or indirect prerequisite in many fundamental research projects, as well as in the design of many practical applications dealing with handwriting. This paper outlines a family of models, derived from the Kinematic Theory of Human Movements. It explains how the nested models in this family can be used coherently, in the context of a multi-level representation paradigm, to analyze both the trajectory and the velocity of strokes with a progressive amount of detail. In the context of a comprehensive survey of previously published work, this paper highlights many new features of stroke production, when the vectorial version of the theory is fully exploited. In this perspective, the Kinematic Theory is depicted as a potential tool to facilitate communications among researchers working in the multi-disciplinary field of Graphonomics.

PsycINFO classification: 2330; 2221; 4140; 2540; 2560

Keywords: Kinematic Theory; Handwriting strokes; Delta–lognormal; Sigma–lognormal; Neuromuscular systems; Rapid-aimed movements

1. Introduction

Handwriting strokes constitute a specific class of rapid human movements, similar to pointing and reaching movements in two dimensions. From the point of view of a

* Corresponding author. Tel.: +1 514 340 4711x4539; fax: +1 514 340 5122.
E-mail addresses: rejean.plamondon@polymtl.ca (R. Plamondon), moussa.djioua@polymtl.ca (M. Djioua).

doi:10.1016/j.humov.2006.07.004

discontinuous representation scheme, they are often considered as basic movement units, the primitives that are used by a writer to produce more complex patterns like a graph, a letter, a word, a signature, etc. They can, in a sense, be considered as a quantum of movements.

Handwriting strokes have been used and studied in many fields of research, and for many reasons. In pattern recognition, for example, numerous algorithms have been designed to recognize handwriting based on the properties of the elementary strokes or primitives that have been used to generate a character or a letter (Plamondon & Srihari, 2000; Plamondon, Lopresti, Schomaker, & Srihari, 1999). In the forensic sciences, a detailed study of individual stroke patterns, focusing on tiny variations, often constitutes the grounds for a decision about the authenticity of a signature (Hilton, 1993; Simner & Girouard, 2000). In education, many teaching methods rely on the production of neat strokes and their concatenation to generate letters and words (Simner, Leedham, & Thomassen, 1996; Wann, Wing, & Søvik, 1991). In motor control, the production of strokes is studied from various points of view (speed-accuracy tradeoffs, optimization principles, fine motor control strategies, developmental coordination, etc.) to gain a better understanding of the underlying cognitive and neuromuscular processes involved in their production (Meulenbroek & Van Gemmert, 2003; Van Galen & Morasso, 1998; Van Gemmert & Teuling, 2004). In the neurosciences, strokes are analyzed to characterize neurodegenerative processes like Parkinson's and Alzheimer's disease (Schröter et al., 2003; Teuling & Stelmach, 1991). They are also used as key patterns to evaluate the recovery processes in the rehabilitation of patients from cerebrovascular accidents (Rohrer et al., 2002). In anthropomorphic robotics, handwriting strokes are used to explore the biomechanical principles employed by humans to produce movements and to apply these concepts to control a robot arm (Potkonjac, 2005). Writing robots are also used to study ink-trace depositions under controlled conditions for forensic signature analysis (Franke & Schomaker, 2004).

Teams focus on various aspects of handwriting strokes, depending on their research goals. Some groups are more interested in their global properties (number, length, curvature, duration, etc.), while others analyze local details like glitches, velocity or acceleration peaks, for example. Still others are more concerned with the residual handwritten trajectory left on a piece of paper, a digitizer or a tablet computer, or with the kinematics or kinetic properties of the strokes.

Many of these studies rely, directly or indirectly, on a stroke generation model. Such a model is used as a cornerstone, a framework on which a research paradigm is built. Many of these models assume that complex patterns like words are made up of simple strokes that are concatenated or superimposed to generate a specific pentip trajectory. In this perspective, a simple stroke is seen as reflecting some fundamental properties of the neuromuscular system of a participant, as well as some basic features of the control strategies that are used to produce a simple movement. The study of individual strokes becomes a specific research theme in and of itself.

Many stroke generation models have been proposed over the years in the context of the various applications previously mentioned (see, for example, Bullock, Grossberg, & Mannes, 1993; Edelman & Flash, 1987; Plamondon & Guerfali, 1998; Plamondon & Maarse, 1989; Wada, Koike, Vatikiotis-Bateson, & Kawato, 2005). In this paper, we present a paradigm to analyze single strokes at various levels of representation, depending on the research goals pursued. Using the Kinematic Theory of Rapid Human Movements (Plamondon, 1995a, 1995b, 1998), we characterize the impulse response of neuromuscular networks with lognormal functions, and use a combination of these functions, as well as various levels of parametric description, to study both the stroke kinematics and its resulting trajectory, either as a function of time or as a spatial 2D image only.

In the next section, we present typical digitizer data to illustrate the great variability displayed by healthy human participants, even in the production of simple strokes. Then, we summarize the main observations that a movement generator has to reproduce in order to mimic these conscious or unconscious behaviors. In Section 3, we provide a brief overview of the Kinematic Theory, and explain how these phenomena can be simulated using lognormal impulse responses to describe the synergetic action of neuromuscular networks. In the following section, we present various simulation results to illustrate our computational approach. In Section 5, the results are discussed in the context of movement control and stroke variability, as well as in the setting of various potential applications. This paper summarizes previously published work under a single framework and, in doing so, highlights several new aspects of the Kinematic Theory, particularly those dealing with the vectorial representation of the input commands and its impact on trajectory direction and curvature computation. Throughout the paper, we add practical comments dealing with the tradeoffs between representation levels, reproduction quality and specific study goals.

2. Stroke variability: Typical experimental results

When a healthy human participant produces a rapid stroke with his dominant hand from a restful posture on a digitizer, the pentip trajectory is characterized by several properties similar to those normally encountered in the production of rapid movements (e.g., Gielen, Van den Oosten, & Van den Pullter, 1985; Latash, 1998; Morasso, 1981; Plamondon, 2003; Zatsiorsky, 1998):

1. The trajectory is nearly straight.
2. There might be up to two reversals in the direction of motion at the end of the trajectory (or one at the beginning and one at the end).
3. The velocity profile might have up to three peaks, the dominant one being slightly asymmetric.
4. The asymmetry of the main velocity peak might be reversed at very high speed.
5. The acceleration profile might have up to four peaks.

If the participant is required to produce a curved stroke, the trajectory is fairly bent but still encompasses the last four of the above properties. Moreover, when a participant repeats the same rapid movement many times, some variability is observed, depending on the participant, but each individual trajectory possesses the aforementioned properties, as long as the movements are fast and there is no trembling or hesitation.

In the next figure, we illustrate some of these basic properties. The examples we give are typical of the numerous data sets collected over the years in our laboratory, using various experimental protocols derived from the following scenario.

A healthy young participant is sitting in a comfortable position, and is required to produce rapid strokes (straight or curved) on a digitizer from a given starting position, with his dominant hand. Some experimental protocols specify conditions like stroke length, required accuracy, minimization of movement time, etc. (see, e.g., Alimi & Plamondon, 1994; Plamondon, Stelmach, & Teasdale, 1990). In some cases, the participant is in a reaction-time experiment, and in others he is given the freedom to produce the required stroke when he is ready to do so. He may be required to voluntarily produce some direction reversals at the beginning or at the end of a single stroke. Sometimes these glitches will just appear in the trajectory on a random basis. In all these experiments, the straight or curved strokes produced by the participants reflect the previously listed properties. The relative frequency of appearance of a specific feature might change from one protocol to another, but what is of interest for the present analysis is the variety of cases observed, since it is these patterns that we want to simulate and analyze here.

Fig. 1 shows typical examples of strokes produced by a human participant when asked to perform a rapid movement. Figs. 1a and 1b show straight strokes, while Figs. 1c and 1d depict curved ones. Under each trajectory, we have plotted the corresponding magnitude of the velocity profiles as computed from digitizer data using time-derivative filters. Unlike the authors of previous papers (e.g., Woch & Plamondon, 2003, 2004), we have not inverted the velocity profiles to illustrate the direction reversals, since we consider this effect from a more general viewpoint here, by taking into account the direction of the velocity vector. We do not present the acceleration profiles either, since they can be directly obtained by time-derivation of the velocity profiles, and, as such, they automatically encompass the properties described in point 5. For example, a two peak smooth

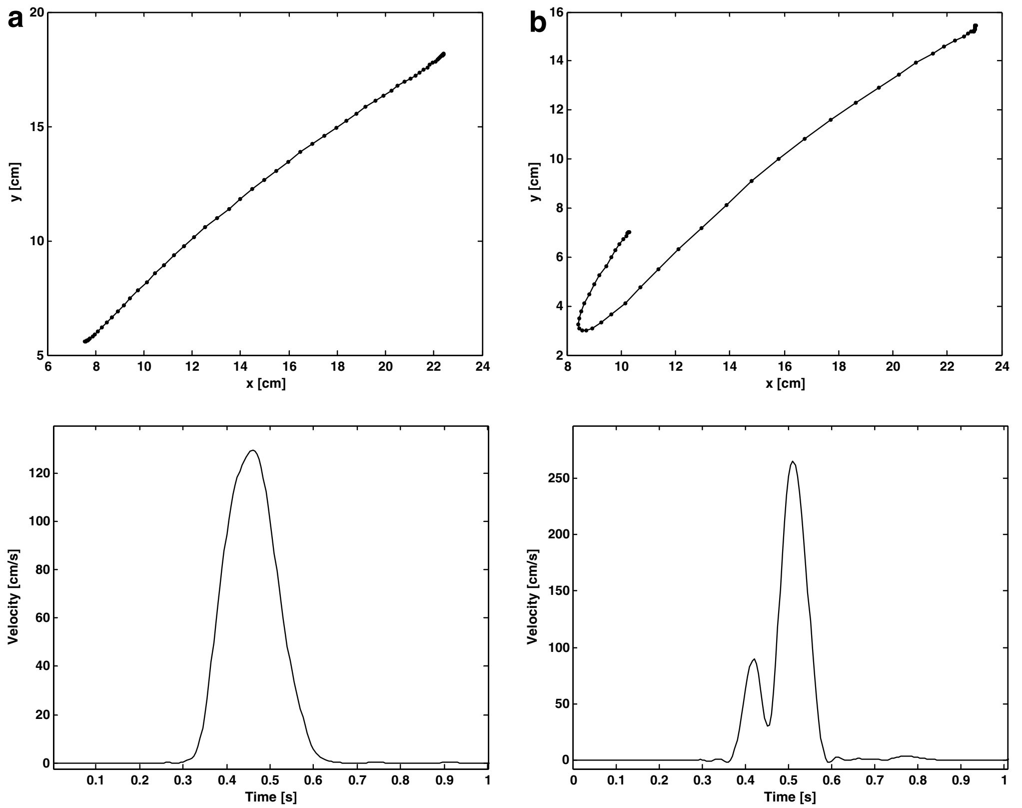


Fig. 1a and 1b. Typical examples of strokes produced by a human participant when performing a rapid movement. Upper graph: straight trajectories; lower graph: velocity profiles.

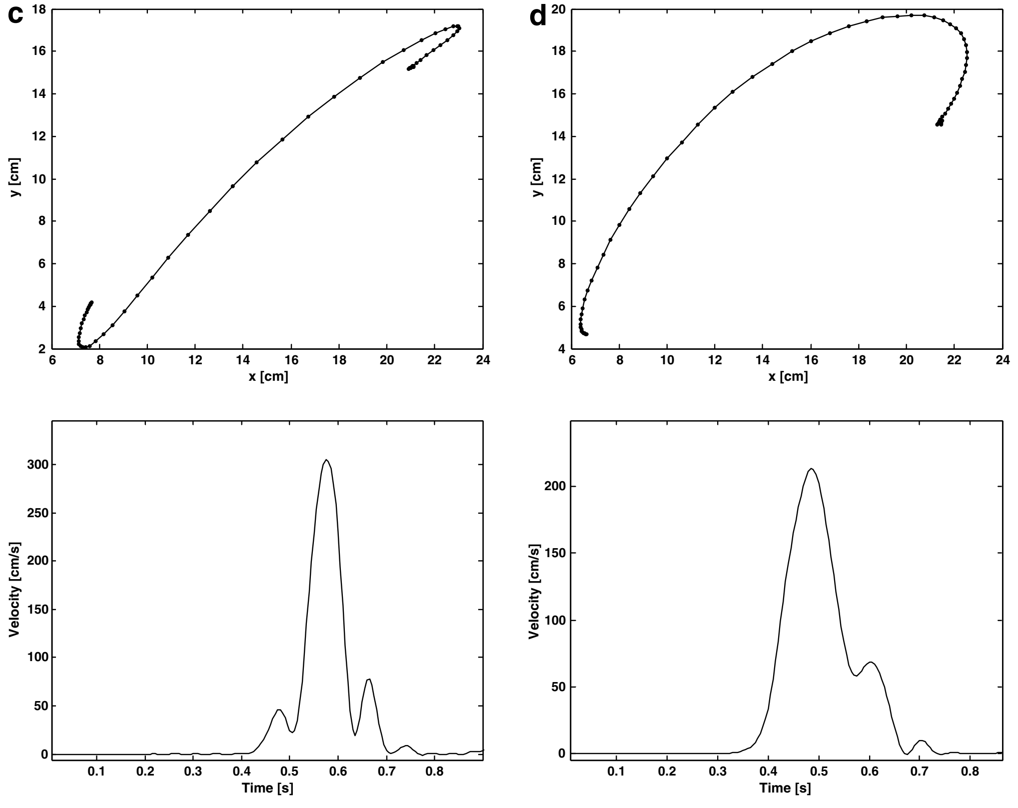


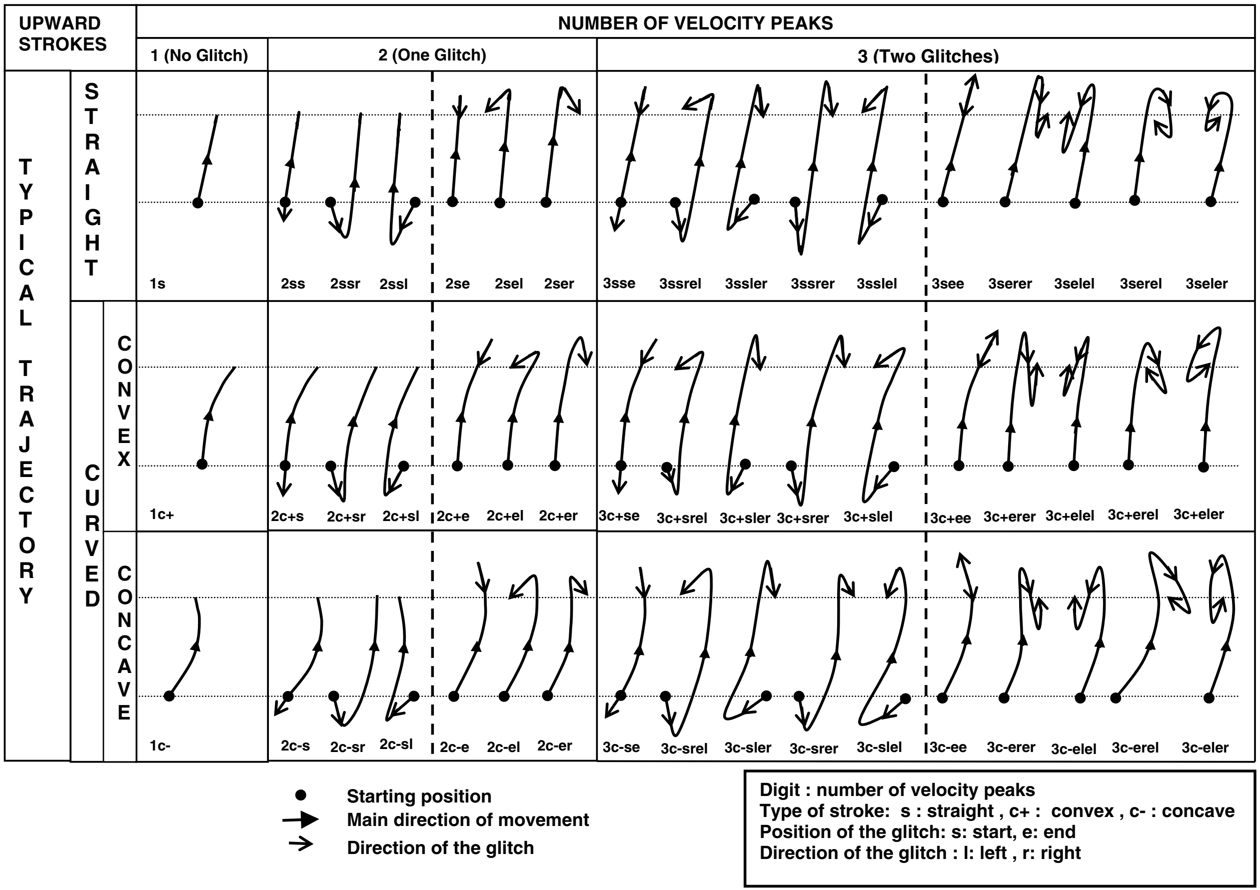
Fig. 1c and 1d. Typical examples of curved strokes produced by a human participant when performing a rapid movement. Upper graph: curved trajectories; lower graph: velocity profiles.

velocity profile leads to a three peak acceleration profile (Plamondon, 1998). The reader must notice that these dynamic features might not be as easily observed when one deals with elderly or motor disabled participants. In these cases, movements are often slow. The velocity profiles are not smooth and this results in acceleration profiles that often have a larger number of peaks.

As one can see, some of the basic features reported in the introduction are clearly illustrated in these typical examples. The trajectory is not perfectly straight (Figs. 1a and 1b) or circular (Figs. 1c and 1d). Some glitches can be observed, and these glitches are not exactly in the opposite direction of the main displacement. Fig. 1a presents a direct trajectory with no glitches and a smooth velocity profile. Fig. 1b depicts an almost straight trajectory with a large glitch at the beginning, its velocity profile being smooth and having two peaks. Figs. 1c and 1d exhibit trajectories with two glitches: a slightly curved trajectory with one glitch at the beginning and one at the end (Fig. 1c), and a curved stroke with two small glitches at the end (Fig. 1d). Both velocity profiles have three peaks in these cases.

In previous papers (Plamondon, 1998, 2003; Woch & Plamondon, 2001, 2003, 2004), we have briefly explained the possible origin of velocity and acceleration variability on perfectly straight strokes using the Kinematic Theory and its delta–lognormal equation.

Table 1
Various types of glitches that can be observed on single straight or curved upward strokes



We have shown, among other things, that properties 3–5 can be explained by the variability associated with the seven parameters of the delta–lognormal equation.

Since this equation describes the magnitude of the velocity vector, it does not provide many cues for the analysis of the variability of the trajectory itself, except for the direction reversals at the beginning or the end of the trajectory (property 2). To analyze directional effects and their variability in more detail, the Kinematic Theory must be used in full, that is, in its vectorial version (Plamondon & Djioua, 2005; Plamondon & Guerfali, 1998).

In the next sections, we use such an approach to depict the various predictions of the Kinematic Theory with respect to stroke trajectory variability. We focus mainly on direction variability, and provide, among other things, some explanations for several phenomena like the departure from straight lines and the presence of glitches, as well as the possibility of generating curved paths.

In Table 1 we have schematically summarized the various types of glitches that can be observed on single straight or curved upward strokes, depending on the number of peaks in their velocity profiles. The starting position is indicated by a black dot and the main direction of movement by a full arrow. An open arrow highlights the direction of a specific glitch. A similar set of patterns would be observed for downward strokes. Moreover, any of these patterns can emerge unintentionally, or can be voluntarily generated by a participant producing a single rapid movement.

The comprehensive reproduction of these patterns is a challenge for any theory or model aimed at describing basic movement primitives in an economical way. How can we explain such trajectory variability and such consistency in the observed variability? These are the fundamental questions that we address in the remainder of this paper.

3. Kinematic Theory

The fundamental idea that constitutes the cornerstone of the Kinematic Theory (Plamondon, 1995a, 1995b, 1998; Plamondon, Feng, & Woch, 2003) is that a neuromuscular system involved in the production of a rapid movement can be considered locally as a linear system made up of a large number of coupled subsystems controlling the velocity of the end effector. The impulse response of such a system converges toward a lognormal function when the number of subsystems is very large and when the coupling between subsystems can be described globally with a proportionality relationship between the cumulative time delays of adjacent subsystems.

In this context, the output of a specific neuromuscular system i is a velocity vector described by

$$\vec{v}_i(t - t_{0i}) = \vec{D}_i \Lambda_i(t; t_{0i}, \mu_i, \sigma_i^2), \tag{1}$$

where

$$\Lambda_i(t; t_{0i}, \mu_i, \sigma_i^2) = \frac{1}{\sigma_i\sqrt{2\pi}(t - t_{0i})} \exp\left\{-\frac{1}{2\sigma_i^2}[\ln(t - t_{0i}) - \mu_i]^2\right\}, \tag{2}$$

$\vec{D}_i$: vector describing the amplitude and the direction of the input command i; t_{0i}: time occurrence of the input command i; μ_i: logtime delay; the time delay of neuromuscular system i on a logarithmic time scale; σ_i: logresponse time; the response time of neuromuscular system i on a logarithmic time scale.

3.1. Delta–lognormal model ($\Delta\Lambda$)

According to the delta–lognormal ($\Delta\Lambda$) paradigm, the production of a rapid stroke requires the synergetic activation of two neuromuscular systems, one agonist and the other antagonist to the direction of the movement. These two systems are simultaneously activated by two input commands $\vec{D}_1$ (agonist) and $\vec{D}_2$ (antagonist) at a time t_0. These synchronous commands propagate in parallel across the two neuromuscular systems, each of which is described by a lognormal impulse response and has its own timing properties.

In this context, the two neuromuscular systems are assumed to react in opposite directions, and the Kinematic Theory predicts that the magnitude of the velocity profile $|\vec{v}(t-t_0)|$ will be described by a delta–lognormal equation (Plamondon, 1995a; Plamondon et al., 2003):

$$|\vec{v}(t-t_0)| = |\vec{D}_1|\Lambda_1(t;t_0,\mu_1,\sigma_1^2) - |\vec{D}_2|\Lambda_2(t;t_0,\mu_2,\sigma_2^2). \tag{3}$$

This model produces rectilinear strokes and predicts all the velocity patterns that can be observed in a set of strokes (Plamondon, 1995a, 1995b). Taking the time derivative of Eq. (3) also leads to acceleration patterns that encompass all the basic features of rapid strokes (Plamondon, 1998).

3.2. Sigma–lognormal model ($\vec{\Sigma}\Lambda$)

The sigma–lognormal ($\vec{\Sigma}\Lambda$) paradigm does not assume that the two neuromuscular systems are working in precisely opposite directions. The output velocity is thus described by a vectorial summation of the contribution of each neuromuscular system involved in the production of a stroke (Plamondon & Guerfali, 1998):

$$\begin{aligned} \vec{v}(t) &= \vec{v}_1(t-t_0) + \vec{v}_2(t-t_0), \\ \vec{v}(t) &= \vec{D}_1\Lambda_1(t;t_0,\mu_1,\sigma_1^2) + \vec{D}_2\Lambda_2(t;t_0,\mu_2,\sigma_2^2), \end{aligned} \tag{4}$$

hereafter called a $\vec{\Sigma}\Lambda$ function (Plamondon & Djioua, 2005). This vectorial model is very general, and is not limited to a single stroke description. It can be used to study complex 2D patterns like handwriting (Plamondon & Guerfali, 1998), as well as 3D movements (Leduc & Plamondon, 2001). In this latter case, the summation is generally made over more than two $\vec{D}_i\Lambda_i$ functions.

There are two versions of the $\vec{\Sigma}\Lambda$ model, the choice depending on the type of the input command used, a straight vector or a curved vector.

3.2.1. Straight input vector

In this simpler version, the input commands $\vec{D}_i$ are considered as standard linear vectors. In other words, the direction of motion is now taken into account by $\vec{D}_1$ and $\vec{D}_2$, the two vectorial input commands, both having an amplitude D_1 or D_2 and a direction θ_1 and θ_2 defined with respect to an arbitrary reference axis.

According to this vectorial paradigm, the magnitude of the velocity profile of a rapid aimed movement is given by

$$|\vec{v}(t)| = \sqrt{D_1^2\Lambda_1^2 + D_2^2\Lambda_2^2 + 2D_1D_2\Lambda_1\Lambda_2\cos(\theta_2-\theta_1)}, \tag{5}$$

where we have used $\Lambda_i = \Lambda_i(t; t_0, \mu_i, \sigma_i^2)$ for simplicity. Moreover, the direction of the velocity as a function of time $\varphi(t)$ is given by

$$\varphi(t) = \mathrm{arctg}\left(\frac{D_1\Lambda_1 \sin\theta_1 + D_2\Lambda_2 \sin\theta_2}{D_1\Lambda_1 \cos\theta_1 + D_2\Lambda_2 \cos\theta_2}\right). \tag{6}$$

It is common practice to analyze a movement with respect to its agonist (required) direction, that is, to define $\theta_1 = 0$. In this context, it can be seen that the $\vec{\Sigma}\Lambda$ function is a generalization of the $\Delta\Lambda$ equation. Indeed, the latter can be directly obtained for the specific case of two opposite input commands, that is, $\theta_2 - \theta_1 = \pi$. In fact, in this condition we have:

$$|\vec{v}(t)| = \sqrt{D_1^2\Lambda_1^2 + D_2^2\Lambda_2^2 - 2D_1D_2\Lambda_1, \Lambda_2} \tag{7}$$

$$= D_1\Lambda_1 - D_2\Lambda_2 \tag{8}$$

$$= \Delta\Lambda \text{ equation}$$

with $\varphi(t) = 0$, a constant in the direction $\theta_1 = 0$ of the agonist movement, D_1, D_2 the amplitudes of the two vectorial commands.

The $\Delta\Lambda$ model predicts a horizontal straight line trajectory under these specific conditions, while the straight $\vec{\Sigma}\Lambda$ model expands these predictions. It can generate quasi-linear trajectories, that is, strokes that are not perfectly linear, as well as some types of curved strokes (Plamondon & Djioua, 2005).

3.2.2. Curved $\vec{\Sigma}\Lambda$ model

A more complex and more precise version of the $\vec{\Sigma}\Lambda$ model can be obtained if it is assumed that the input command vectors are not straight, but curved. Their magnitude is still described by $D_i = |\vec{D}_i|$, but the vectors are described by circular arcs of constant curvature. The direction of these vectors evolves over time. This evolution can be taken into account in two equivalent ways: either by working in terms of the vector curvature or in terms of the starting and ending directions of the circular arc vector.

In the first case, Eq. (6) can be expressed by (Plamondon & Guerfali, 1998)

$$\varphi_i(t) = \theta_i + C_i \int_{t_0}^{t} v(\tau)\,\mathrm{d}\tau, \tag{9}$$

where θ_i is the initial direction of the ith input command and C_i describes the curvature of the ith circular arc. Using the relationship

$$C_i = \frac{\theta_{ie} - \theta_{is}}{D_i}, \tag{10}$$

where θ_{is} is the starting direction of the curved input command and θ_{ie} the ending direction of the curved input command, then $\varphi_i(t)$ can be expressed by (Djioua, 2006)

$$\varphi_i(t) = \theta_{is} + \frac{\theta_{ie} - \theta_{is}}{D_i} \int_{t_{0i}}^{t} v_i(\tau)\,\mathrm{d}\tau. \tag{11}$$

These two paradigms describe a more general control of the individual stroke curvature by incorporating it directly into the vectorial model. Such a model can generate the largest variety of strokes (Djioua, 2006; O'Reilly, Djioua, & Plamondon, 2005).

4. Stroke generation

All the previous models derived from the lognormal paradigm of the Kinematic Theory can be seen as simplified versions of the curved $\vec{\Sigma}\Lambda$ model. Indeed, the straight $\vec{\Sigma}\Lambda$ model can be obtained if the following constraints are put on the curved model:

$$\begin{aligned} \theta_{1s} &= \theta_{1e} = \theta_1 = \text{constant}, \\ \theta_{2s} &= \theta_{2e} = \theta_2 = \text{constant} \end{aligned} \tag{12}$$

and, as we have seen, the $\Delta\Lambda$ model can be derived from the straight $\vec{\Sigma}\Lambda$ model, if

$$\theta_2 - \theta_1 = \pi.$$

Using the curved $\vec{\Sigma}\Lambda$ model, the Kinematic Theory can be used to generate any single stroke with almost any required precision, depending on the number of parameters that are used in the computer simulations.

In the examples below, we have used the following relationship to reduce the number of parameters:

$$\begin{aligned} \theta_{2s} &= \theta_{1e} + \pi \pm \Delta\theta, \\ \theta_{2e} &= \theta_{1s} + \pi \pm \Delta\theta \end{aligned} \tag{13}$$

that is, we have assumed that the starting antagonist direction θ_{2s} is in opposition ($+\pi$) to the ending agonist direction θ_{1e}, plus or minus some variability factor ($\pm\Delta\theta$). Similarly, the ending antagonist direction θ_{2e} was considered to be in opposition ($+\pi$) to the starting agonist direction θ_{1s}, plus or minus the same variability factor ($\pm\Delta\theta$).

Fig. 2 depicts strokes with a single velocity peak: Fig. 2a straight (1s), Fig. 2b curved (convex: 1c+) and Fig. 2c curved (concave: 1c−) strokes. Fig. 3 presents the examples of strokes with two velocity peaks, one from each group of the list schematized in Table 1 (Fig. 3a straight: 1ssl, Fig. 3b convex: 1c + el and Fig. 3c concave: 1c − sl). Figs. 4 and 5 deal with strokes having three peaks. Figs. 4a–c present specimens with one small peak at the beginning and at the end of the stroke, while Figs. 5a–c depict strokes with two small peaks after the main peak. Here again, Figs. 4a and 5a show straight strokes (3sslel, 3serel), while the examples in Figs. 4b, 5b and 4c, 5c show convex (3c + sler, 3c + eler) and concave (3c − slel, 3c − erer) strokes respectively, following the nomenclature of Table 1. Table 3, presented in the appendix, gives the list of specific parameters that have been used for each simulation.

Extrapolating from these examples, the Kinematic Theory can take into account all the basics patterns that can be observed on a single stroke, and, as such, can be used to generate human-like strokes (cf. Fig. 1), both in the kinematics and the static representation space. In the next section, we discuss this point theoretically, in terms of motor control, and, practically, in terms of the trade-off between the representation power of the theory versus the number of parameters involved.

5. Discussion

The interest in using the Kinematic Theory to study handwriting strokes is that the same paradigm can be used whatever the detail level needed for a specific application. It provides a unified point of view in terms of movement description and understanding.

5.1. Theoretical aspects

From a motor control perspective, the theory assumes that well-learned movements are produced by the synergetic activation of neuromuscular systems, each system being described by a lognormal impulse response, that is, each system has a specific logtime delay

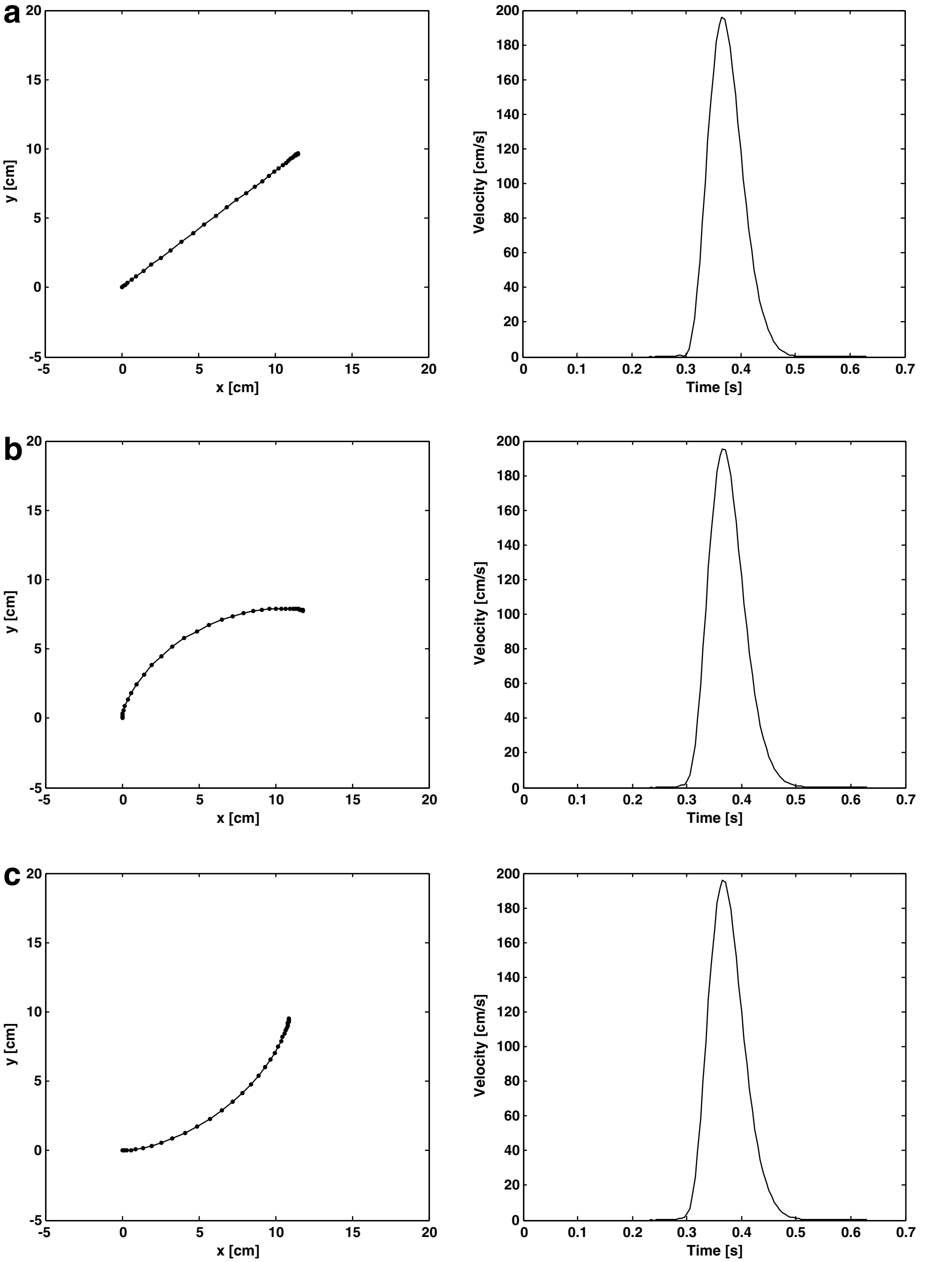


Fig. 2. Typical examples of simulated strokes with a single peak velocity profile.

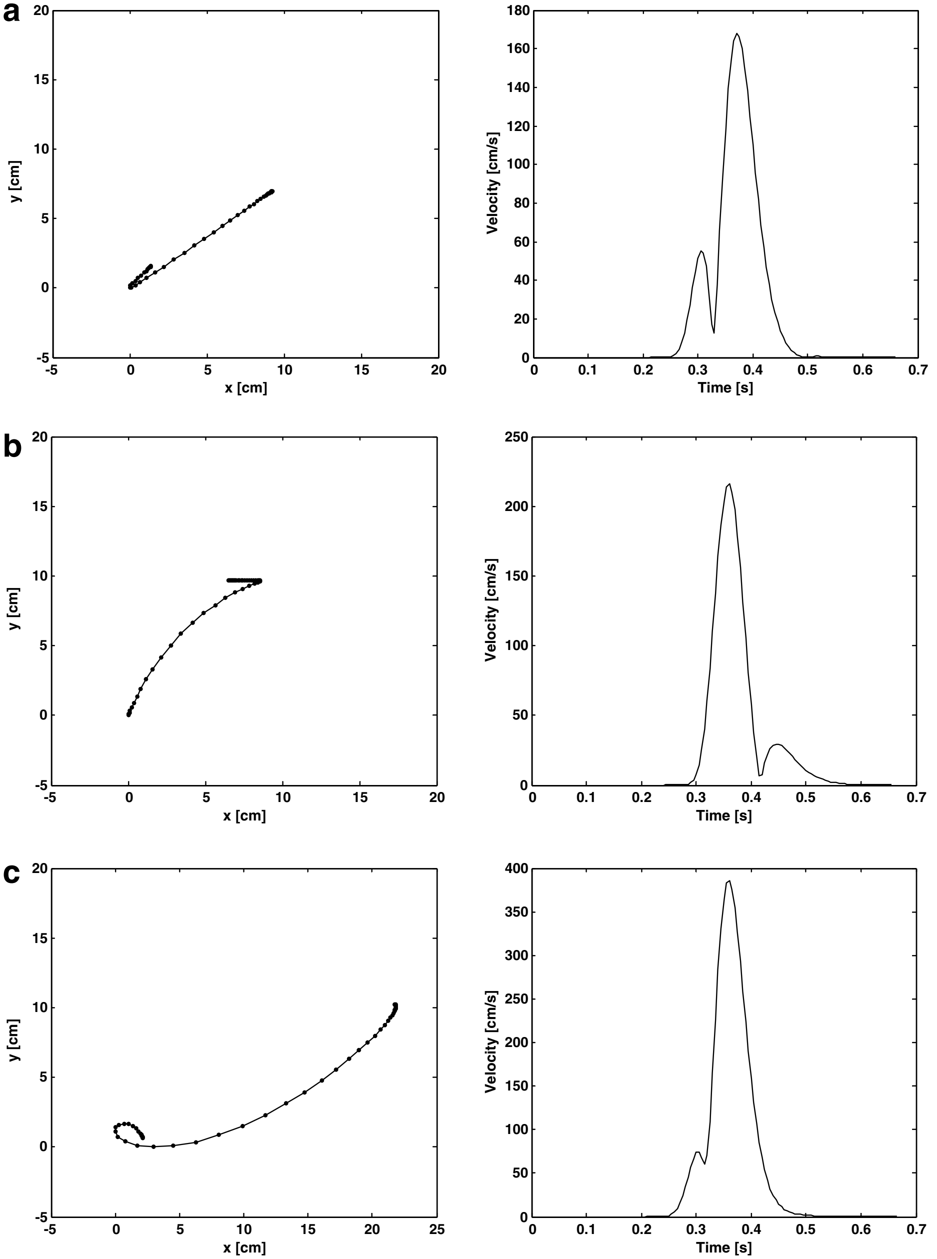


Fig. 3. Typical examples of simulated strokes with two peaks in their velocity profiles.

μ and logresponse time σ. An individual system is activated via a vectorial input command $\vec{D}$ occurring at a time t_0. A simple handwriting stroke requires the activation of two neuromuscular systems (agonist/antagonist) with two synchronous input commands, and the resulting movement is controlled in the velocity domain. The end effector (pentip) velocity is the vectorial summation of the contribution of each neuromuscular system to the synergy.

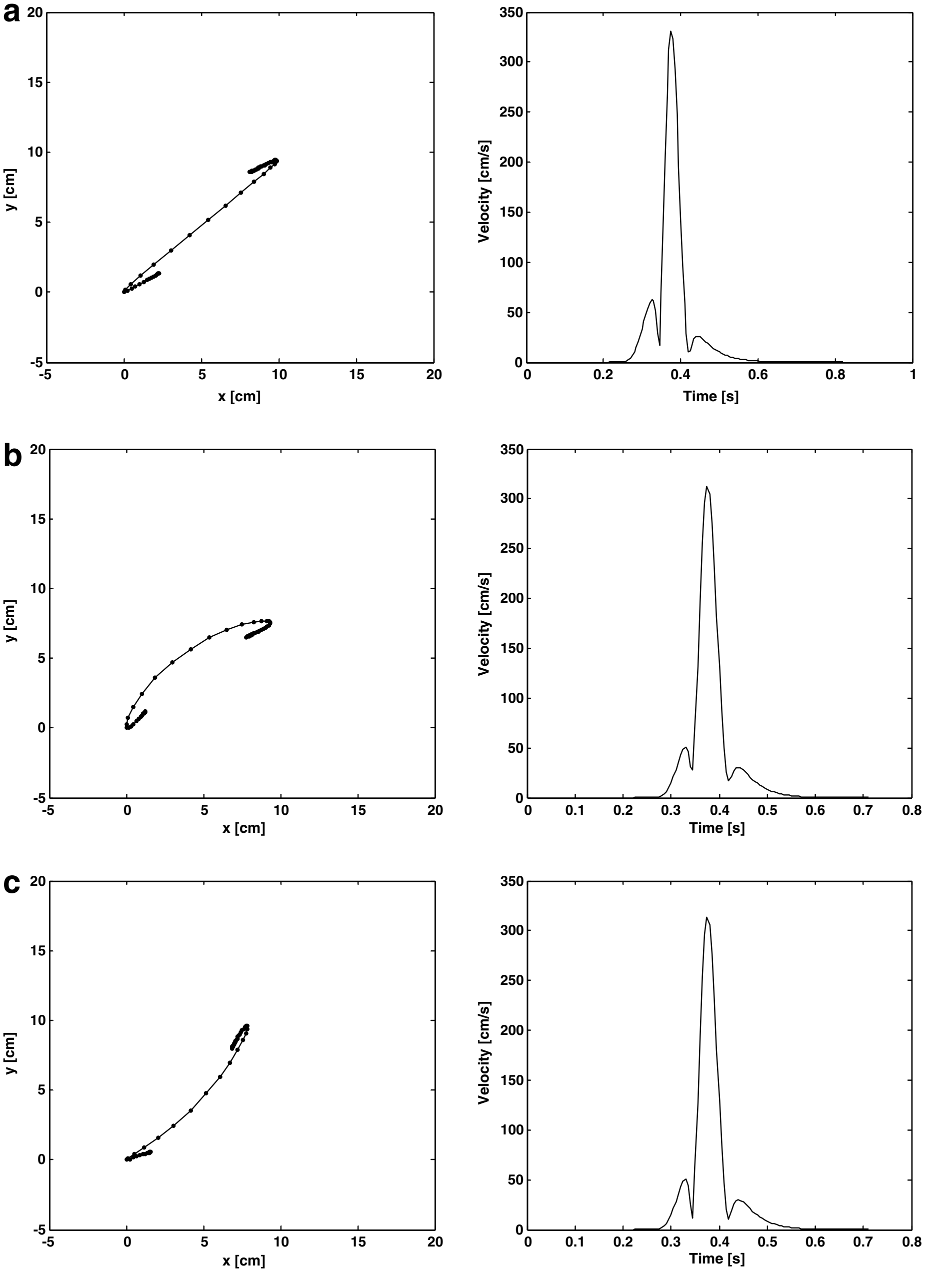


Fig. 4. Typical examples of simulated strokes of three-peak velocity, one small peak before the main peak and one after it.

In other words, to produce a handwriting stroke, a participant has to prepare an action plan made up of two input commands $\vec{D}_1$ and $\vec{D}_2$. These commands, which might be associated with the activity of cell populations in the central nervous system, are fed

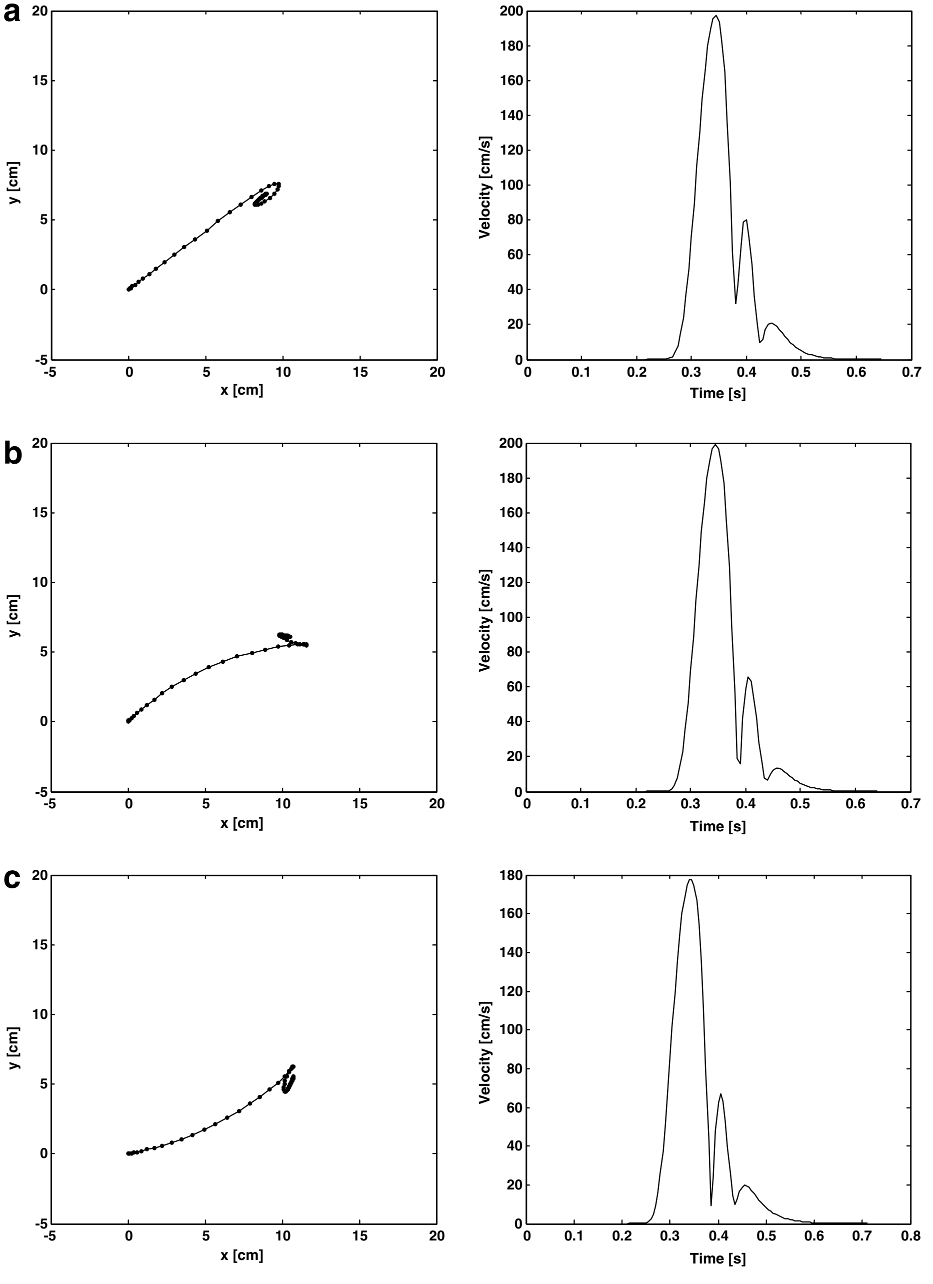


Fig. 5. Typical examples of simulated strokes with three peak velocity profiles, that is, two small peaks after the main peak.

into the agonist and antagonist neuromuscular systems that react to these within the logtime delay (μ_1, μ_2) and the logresponse times (σ_1, σ_2) of their respective impulse responses.

Table 2
The various representation levels of a single stroke, as described by the Kinematic Theory

Model	Parameters	Number of parameters	Comments
1. Curved sigma–lognormal	Command: D_1, θ_{1s}, θ_{1e}, D_2, θ_{2s}, θ_{2e}, t_0 System: μ_1, μ_2, σ_1, σ_2	11	The most general curved stroke generation model
2. Curved sigma–lognormal with direction constraint	Command: D_1, θ_{1s}, θ_{1e}, D_2, $\Delta\theta$, t_0 System: μ_1, μ_2, σ_1, σ_2	10	Antagonist direction linked to the agonist direction $\theta_{2s} = \theta_{1e} + \pi \pm \Delta\theta$, $\theta_{2e} = \theta_{1s} + \pi \pm \Delta\theta$
3. Curved sigma–lognormal with curvature constraint	Command: D_1, θ_{1s}, D_2, C_0, t_0 System: μ_1, μ_2, σ_1, σ_2	9	Link between curvature and direction $C_0 = (\theta_{ie} - \theta_{is})/D_i$
4. Linear sigma–lognormal	Command: D_1, θ_{1s}, D_2, θ_{2s}, t_0 System: μ_1, μ_2, σ_1, σ_2	9	The most general linear stroke generation model
5. Minimal sigma–lognormal	Command: D_1, θ_{1s}, D_2, θ_{2s}, t_0	5	The simplest $\vec{\Sigma}\Lambda$ model: the system parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2)$ are fixed to some realistic values
6. Delta–lognormal	Command: D_1, D_2, t_0 System: μ_1, μ_2, σ_1, σ_2	7	Perfect agonist/antagonist opposition $\theta_{2s} - \theta_{1s} = \pi, \quad \theta_{2e} = \theta_{2s}, \quad \theta_{1e} = \theta_{1s}$
7. Minimal delta–lognormal	Command: D_1, D_2, t_0	3	The simplest $\Delta\Lambda$ model: the system parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2)$ are fixed to some realistic values

The complexity of the model is directly linked to the number of parameters used to describe the input commands. We have summarized in Table 2 the various representation levels that are available, in descending order of complexity. The most general model, the curved $\vec{\Sigma}\Lambda$, requires 11 parameters: four system parameters, six command parameters, and t_0, the time occurrence of the synchronous activation of the commands. According to this paradigm, the action plan is made up of curved vector commands that are specified by their magnitude (D) and their curvature as expressed in terms of the starting (θ_s) and the ending (θ_e) directions of the vectors. Before making a stroke, a participant has had to estimate the stroke length as well as its curvature. This model can reproduce all the stroke patterns of Table 1. The other two versions of this model (rows 2 and 3) are computational tricks for reducing the number of parameters of the system by integrating some constraints between the agonist/antagonist directions (row 2) or between the curvature and the direction (row 3). The third model is almost as powerful as the first two, except that the glitches at the beginning and the end of a stroke are always located on the same side (start-right and end-left, or start-left and end-right).

Such a model can be used to describe a single, joint movement, while the first two are more appropriate to the study of multi-joint movements.

The linear $\vec{\Sigma}\Lambda$ model (row 4) works with rectilinear input commands. Only the initial direction of the command is necessary to produce a realistic quasi-linear stroke with various glitches as well as curved strokes, but, in this latter case, trajectories do not have glitches. A simplified version of this model (row 5) assumes that the μ_i, σ_i are fixed for a given type of movement made by a stable synergy. These two models are appropriate, for example, to study the variability of rectilinear strokes.

The Delta–lognormal model (row 6) is a linear $\vec{\Sigma}\Lambda$ with perfect agonist/antagonist opposition. The model produces straight strokes which can encompass glitches that are in the reverse direction of the movement. A simplified version of this model (row 7), assumes that the μ_i, σ_i are fixed to some realistic values. These latter two models are generally sufficient to study the velocity patterns associated with a single stroke. The straighter the trajectory, the better the description.

5.2. Variability

As one can see, the variability observed in the production of a set of similar strokes by a participant can come from many sources, those being the various parameters used in a given model. At the command level, fluctuations in the magnitude of the input commands will affect stroke length ($D_1 - D_2$) and stroke duration (linked to D_1/D_2) (Plamondon, 1995b). They will also influence the relative importance of the agonist and antagonist contribution to a stroke, and they can produce directional change at both ends of a stroke. These glitches will not occur in the main orientation of the movement, if there are fluctuations in direction θ_i of the orientation of the input commands. However, the changes that they generate, combined with fluctuations in the parameters μ_i and σ_i, will result in variations in the velocity profiles, as well as in departures from perfectly straight or circular strokes. It must be noted, however, that a variation in t_0 will not affect either the kinematics or the static aspects of a trace, but will introduce a time translation with respect to some time stamp, as defined, for example, in a reaction time paradigm.

Moreover, these glitches can be produced voluntarily (sometimes with a little bit of practice) or unintentionally, depending on the fluency of the handwriting. In neither case are the glitches considered as distinct strokes, however, since they are generated from a

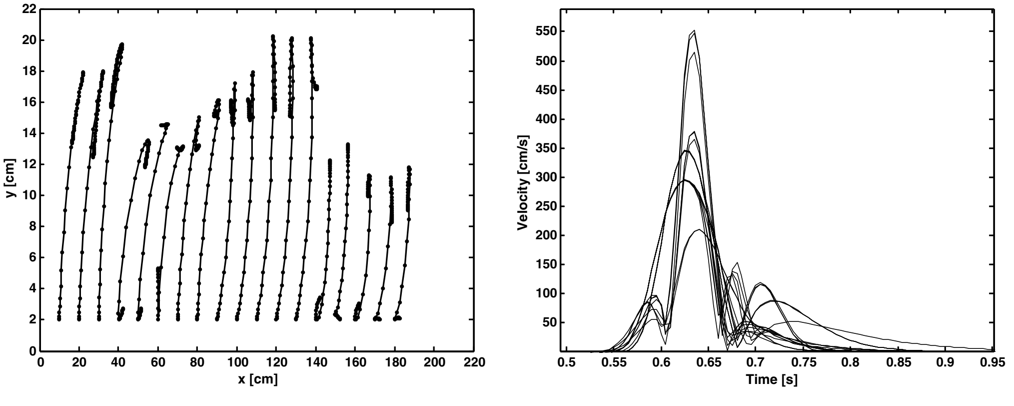


Fig. 6. A series of strokes generated from an original set of parameters to which random variations have been added. The original parameter values were $t_0 = 0.5$, $D_1 = 26.4$, $\mu_1 = -2$, $\sigma_1 = 0.22$, $\theta_{s1} = 74.88$, $\theta_{e1} = 12.96$, $D_2 = 13.4$, $\mu_2 = -1.56$, $\sigma_2 = 0.22$, $\theta_{s2} = -176.3$, $\theta_{e2} = -133$.

single pair of synchronous input commands $\vec{D}_1$ and $\vec{D}_2$ occurring at t_0, and activating a synergy made up of an agonist and an antagonist lognormal neuromuscular system. They are thus different from what is often referred to as secondary strokes. In this latter case, at least a second set of input commands is necessary to generate these distinct submovements. In other words, the Kinematic Theory provides an efficient criterion for parsing individual strokes, each of which might incorporate up to two glitches, from a series of concatenated strokes. A single stroke is defined as any movement that results from the synchronous activation of two competing lognormal neuromuscular systems.

We have illustrated some of these phenomena in Fig. 6. All the strokes and velocity profiles depicted here were simulated from a single set of parameters to which random variations were added. In other words, every parameter of every curve was assumed to be normally distributed around a mean value, and each curve was generated using a random set of values. The trajectories have been slightly translated horizontally to highlight the small differences, while the velocity profiles have been superimposed on the same graph to highlight the similarities. This mimics, for example, how a participant, being asked to produce a specific set of identical strokes, would react to the instructions given.

5.3. Practical issues

As expected, the quality of the kinematic and static description of a stroke is directly linked to the number of parameters used in a given model. As mentioned in the introduction, depending on the application, a specific level of representation will be sufficient. Nice letter models can be generated with three parameters per stroke, and, in this case, the minimal Delta–lognormal model is often sufficient (Djeziri, Guerfali, Plamondon, & Robert, 2002). Studies which focus on velocity patterns only can be run with the full $\Delta\Lambda$ model if the strokes are straight and only the direction reversals have to be taken into account (Guerfali & Plamondon, 1998). If there is also interest in analyzing departures from straight strokes, the linear $\vec{\Sigma}\Lambda$ model will be the best choice (Plamondon & Djioua, 2005). A full study of either straight or curved strokes requires the curved $\vec{\Sigma}\Lambda$ model.

In this context, the use of an increasing number of parameters to obtain a more detailed description of a given stroke should not be seen as an overfitting process. The glitches, the variations in directions and the departure from a perfectly straight or curved path all reflect specific fluctuations at the command or execution level of a stroke. Some of these phenomena might be a specific feature to observe in forensic document analysis or a burdensome noise to filter out in handwriting recognition, or, in other studies, similar variations might characterize specific developmental or neurodegenerative processes. What we have highlighted in this paper is that any of these various points of view can be taken into account using the same theory and a family of movement generation models that allows various levels of movement representation within a unified neuromuscular paradigm.

This approach can be easily used, qualitatively, by conducting a visual inspection of the various strokes produced under a given set of experimental conditions to group together various sets of strokes for subsequent analysis. For example, strokes that have more than two glitches or more than three peak velocity profiles cannot be considered as a single stroke. According to the Kinematic Theory, they are not single basic movements resulting from the synchronous activation of two neuromuscular systems with two synchronous vectorial commands. A similar qualitative approach can also serve as a method to rapidly

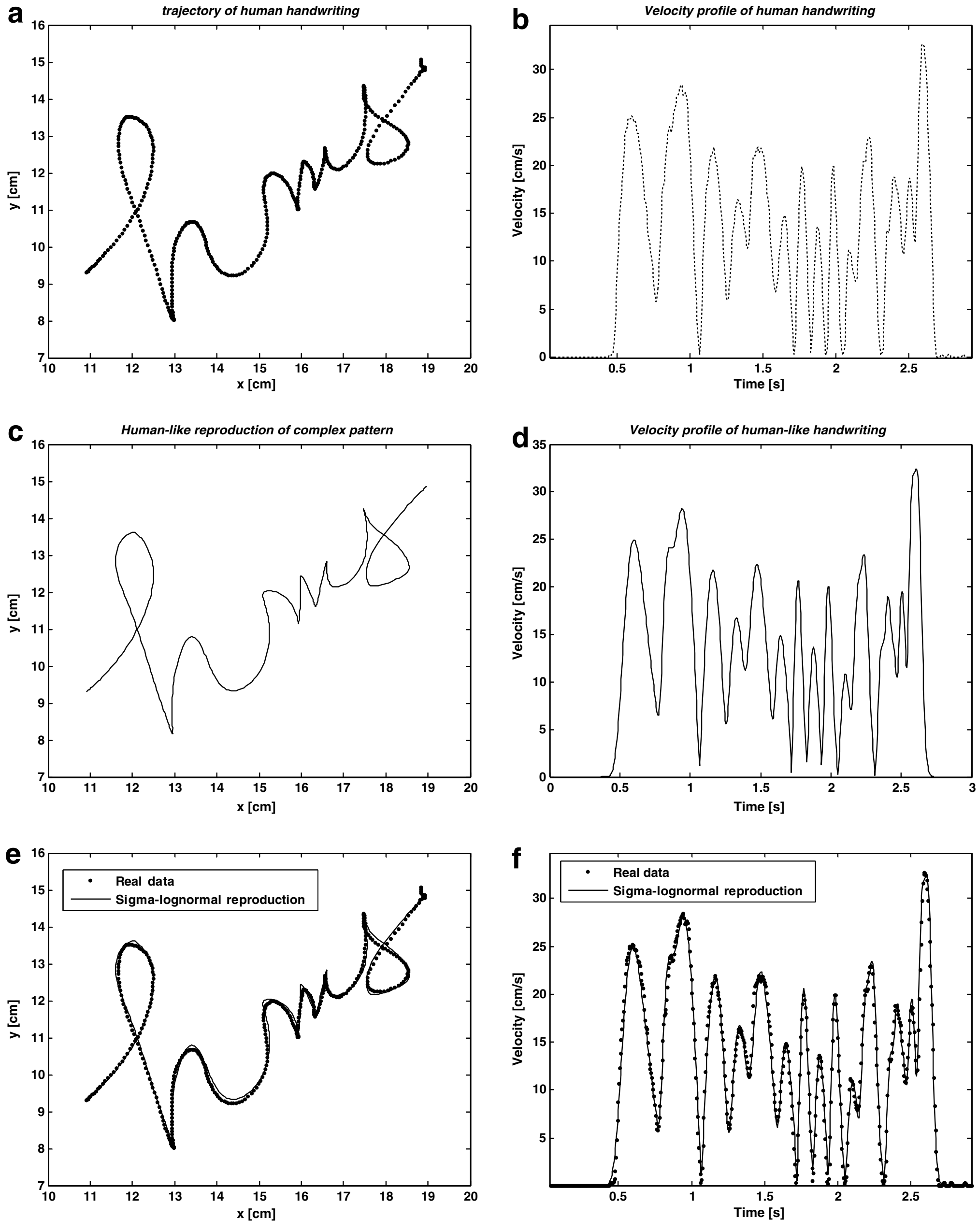


Fig. 7. Illustration of (a) the trajectory of a complex pattern written by a human participant and (b), its corresponding velocity profile. (c) and (d) A human-like reproduction of the trajectory and the velocity profile using the Sigma–lognormal model. Figures (e) and (f), pair wise superimposition of the patterns.

check whether or not a participant meets the task requirements, as well as a way to improve an experimental protocol using more specific instructions for the production of specific strokes.

From a movement synthesis perspective, the theory offers a full mathematical description of strokes that can be used directly in many applications. For example, it has been used recently (Varga, Kilchhofer, & Bunke, 2005) to generate a large database of words and sentences to train handwriting recognition algorithms. The design of such algorithms is very complex, and the larger the database to train and test a system, the better the results and the more robust the system designed. The use of realistic, humanlike patterns automatically generated from the Kinematic Theory using randomly generated sets of parameters can save time and money.

In Fig. 7, we have illustrated the capacity of the $\vec{\Sigma}\Lambda$ model to generate humanlike handwriting. Figs. 7a and b depict a complex pattern, a sequence of letters (hms), written by a human participant (Fig. 7a, trajectory; Fig. 7b, velocity profile), while Figs. 7c and d present a typical reproduction of the same sequence as created with an interactive software tool based on the model (Djioua, O'Reilly, & Plamondon, 2006). In Figs. 7e and f, we have superimposed the real and reconstructed patterns to clearly highlight what is meant by mimicking human writers. The whole pattern was reconstructed with a trajectory MSE of 0.0066 cm^2 and a velocity MSE of 1.1396 cm^2/s^2. The parameter values used for this example are given in the Appendix (Table 4).

Finally, the full benefit of this methodology requires the quantitative analysis of individual strokes, using, for example, nonlinear regression algorithms. The design of such algorithms is very tricky. We have already proposed some efficient methods to process $\Delta\Lambda$ profiles (Djioua, Plamondon, Della Cioppa, & Marcelli, 2005; Guerfali & Plamondon, 1995) and the analysis of $\vec{\Sigma}\Lambda$ patterns is under development (Djioua, 2006).

6. Conclusion

We have explained in this paper how the Kinematic Theory of Rapid Human Movements can be used to generate a family of handwriting stroke generation models. Each of these models takes advantage of the global description of the impulse responses of neuromuscular systems using lognormal functions. Depending on the relatively detailed representation of the input commands, various levels of description can be obtained for a single stroke. In this context, the Kinematic Theory can be used qualitatively or quantitatively to study the production of rapid movements in a variety of fundamental studies and practical applications using a coherent and unified paradigm. It is expected that such a framework will serve to, among other things, facilitate communication and understanding between various research teams directly or indirectly involved in the study of rapid human movements in general, since the overall approach is not limited to handwriting studies.

Acknowledgment

This work was supported by NSERC grant RGPIN-915 to Réjean Plamondon.

Appendix A

Tables 3 and 4.

Table 3
List of the parameters used in the simulations presented in Figs. 2–5 (distances are expressed in centimeters, time in seconds and angles in degrees)

Figure	t_0	D_1	μ_1	σ_1	θ_{s1}	θ_{e1}	D_2	μ_2	σ_2	θ_{s2}	θ_{e2}
Fig. 2a	0.2	20	−1.761	0.192	40	40	5	−1.77	0.237	220	220
Fig. 2b	0.2	20	−1.761	0.192	4.32	65.52	5	−1.77	0.237	219.6	219.6
Fig. 2c	0.2	20	−1.761	0.192	65.52	15.84	5	−1.77	0.237	219.6	219.6
Fig. 3a	0.2	20	−1.761	0.192	40	40	10.6	−1.914	0.289	225	225
Fig. 3b	0.2	22.2	−1.794	0.178	30.96	15.84	11.2	−1.971	0.304	149.04	−246.96
Fig. 3c	0.2	20	−1.761	0.192	−19.44	65.52	10.6	−1.578	0.203	175.68	156.24
Fig. 4a	0.2	20	−1.728	0.111	40	40	11	−1.761	0.324	208.8	213.12
Fig. 4b	0.2	20	−1.728	0.111	12.96	65.52	11	−1.719	0.266	206.64	231.84
Fig. 4c	0.2	20	−1.728	0.111	65.52	29.52	11	−1.719	0.266	245.52	197.28
Fig. 5a	0.2	20	−1.851	0.264	40	40	8.8	−1.632	0.095	208.08	238.32
Fig. 5b	0.2	20	−1.851	0.262	−18.72	47.52	8	−1.599	0.1	153.36	186.48
Fig. 5c	0.2	20	−1.851	0.297	69.84	11.52	8	−1.599	0.1	259.2	223.92

Table 4
List of the parameters used in the simulations presented in Fig. 7 (distances are expressed in centimeters, time in seconds and angles in degrees)

#Lognormal	t_0	D	μ	σ	θ_s	θ_e
1	0.321	5.399	−1.183	0.294	98.64	43.92
2	0.495	0.679	−1.457	0.215	212.76	174.24
3	0.595	1.946	−1.4	0.149	309.6	224.28
4	0.6	5.087	−1.015	0.203	287.64	282.96
5	0.804	4.797	−0.979	0.22	61.56	104.04
6	0.826	3.686	−0.677	0.138	280.8	273.96
7	1.014	4.265	−0.762	0.146	118.44	42.84
8	1.421	1.849	−1.485	0.195	265.68	351
9	1.523	1.559	−1.388	0.116	92.16	83.88
10	1.616	1.076	−1.329	0.118	283.32	297.72
11	1.723	1.366	−1.329	0.102	77.04	81.36
12	1.937	0.931	−1.738	0.198	294.12	264.96
13	1.959	1.704	−1.412	0.138	94.32	27
14	1.999	0.931	−1.365	0.097	105.48	83.88
15	2.105	0.713	−1.365	0.086	313.92	280.8
16	2.158	0.931	−1.365	0.086	313.92	323.64
17	2.202	0.545	−1.365	0.086	313.92	278.64
18	2.243	1.398	−1.314	0.105	197.64	208.44
19	2.337	1.318	−1.232	0.081	46.08	65.52
20	2.391	1.994	−1.664	0.142	39.24	92.52

References

Alimi, A., & Plamondon, R. (1994). Analysis of the parameter dependence of handwriting generation models on movements characteristics. In C. Faure, G. Lorette, A. Vinter, & P. Keuss (Eds.), *Advances in handwriting and drawing: A multidisciplinary approach* (pp. 363–378). Paris: Europa.

Bullock, D., Grossberg, S., & Mannes, C. (1993). A neural network model for cursive script production. *Biological Cybernetics, 70*, 15–28.

Djeziri, S., Guerfali, W., Plamondon, R., & Robert, J. M. (2002). Learning handwriting with a pen-based system: Computational issues. *Pattern Recognition, Special Issue of Handwriting Processing and Applications, 35*, 1049–1057.

Djioua, M., Plamondon, R., Della Cioppa, A., & Marcelli, A. (2005). Delta–lognormal parameter estimation by non-linear regression and evolutionary algorithm: A comparative study. In *Proceedings of the 12th international conference on graphonomics society* (pp. 44–48). University of Salerno, Italy.

Djioua, M., O'Reilly, C., & Plamondon, R. (2006). An interactive trajectory synthesizer to study outlier patterns in handwriting recognition and signature verification. In *Proceedings of the 18th conference on pattern recognition, Hong Kong*, in press.

Djioua, M. (2006). Analyse et synthèse des mouvements humains par l'utilisation du modèle sigma–lognormal. PhD thesis, École Polytechnique de Montréal.

Edelman, S., & Flash, T. (1987). A model of handwriting. *Biological Cybernetics, 57*, 25–36.

Franke, K., & Schomaker, L. R. B. (2004). Robotic writing trace synthesis and its application in the study of signature line quality. *Journal of Forensic Document Examination, 16*, 119–146.

Gielen, C. C. A. M., Van den Oosten, K., & Van den Pullter, G. F. (1985). Relation between EMG activation patterns and kinematic properties of aimed arm movement. *Journal of Motor Behaviour, 17*, 421–442.

Guerfali, W., & Plamondon, R. (1995). Signal processing for the parameter extraction of the delta–lognormal model ($\Delta\Lambda$). In C. Archibald & P. Kwok (Eds.), *Research in computer and robot vision* (pp. 217–232). Singapore: World Scientific.

Guerfali, W., & Plamondon, R. (1998). New method for the analysis of simple and complex planar rapid movements. *Journal of Neuroscience Methods, 82*, 35–45.

Hilton, O. (1993). Scientific examination of questioned documents. In *CRC series in forensic and police science*. CRC Press, Boca Raton, London, revised edition.

Latash, M. L. (1998). *Neurophysiological basis of movement*. Champaign, IL: Human Kinetics.

Leduc, N., & Plamondon, R. (2001). A new approach to study human movements: The three dimensional delta–lognormal model. In *Proceedings of the 10th biennial conference of the international graphonomics society* (pp. 98–102), University of Nijmegen, The Netherlands.

Meulenbroek, R. G. J., & Van Gemmert, A. W. A. (Eds.) (2003). Advances in the study of drawing and handwriting [Special issue]. *Human Movement Science, 22*, 131–220.

Morasso, P. (1981). Spatial control of arm movements. *Experimental Brain Research, 42*, 223–227.

O'Reilly, C., Djioua, M., & Plamondon, R. (2005). Système de synthèse de l'écriture manuscrite par l'utilisation du modèle sigma–lognormal. Technical Report EPM-/RT-2-5-07, École Polytechnique de Montréal.

Plamondon, R. (1995a). A kinematic theory of rapid human movements. Part I: Movement representation and generation. *Biological Cybernetics, 72*, 295–307.

Plamondon, R. (1995b). A kinematic theory of rapid human movements. Part II: Movement time and control. *Biological Cybernetics, 72*, 309–320.

Plamondon, R. (1998). A kinematic theory of rapid human movements. Part III: Kinetic outcomes. *Biological Cybernetics, 78*, 133–145.

Plamondon, R. (2003). A unified framework to study human movements under isotonic and isometric conditions. *Neuroscience*, New Orleans, USA.

Plamondon, R., & Djioua, M. (2005). Handwriting stroke trajectory variability in the context of the kinematic theory. In *Proceedings of the international graphonomics society* (pp. 250–254), University of Salerno, Italy.

Plamondon, R., Feng, C., & Woch, A. (2003). A kinematic theory of rapid human movement. Part IV: A formal mathematical proof and new insights. *Biological Cybernetics, 89*, 126–138.

Plamondon, R., & Guerfali, W. (1998). The generation of handwriting with delta–lognormal synergies. *Biological Cybernetics, 78*, 119–132.

Plamondon, R., Lopresti, D., Schomaker, L. R. B., & Srihari, R. (1999). On-line handwriting recognition. In J. G. Webster (Ed.). *Wiley encyclopedia of electrical and electronics engineering* (15, pp. 123–146). NY: John Wiley & Sons.

Plamondon, R., & Maarse, F. J. (1989). An evaluation of motor models of handwriting. *IEEE Transactions on System, Man and Cybernetics, 19*, 1060–1079.

Plamondon, R., & Srihari, S. (2000). On-line and off-line handwriting recognition: A comprehensive survey, 20th Anniversary Special Issue. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 22*, 63–84.

Plamondon, R., Stelmach, G. E., & Teasdale, N. (1990). Motor program coding representation from handwriting generator model: The production of lines responses. *Biological Cybernetics, 63*, 443–451.

Potkonjac, V. (2005). Robotic handwriting. *International Journal of Humanoid Robotics, 2*, 105–124.

Rohrer, B., Fasoli, S., Krebs, H. I., Hughes, R., Volpe, B., & Frontera, W. R. (2002). Movement smoothness changes during stroke recovery. *The Journal of Neuroscience, 22*, 8297–8304.

Schröter, A., Mergl, R., Bürger, K., Hampel, H., Möller, H.-J., & Hegerl, U. (2003). Kinematic analysis of handwriting movements in patients with Alzheimer's disease, mild cognitive impairment, depression and healthy subjects. *Dementia and Geriatric Cognitive Disorders, 15*, 132–142.

Simner, M. L., & Girouard, P. L. (Eds.) (2000). Advance in forensic document examination [Special issue]. *Journal of Forensic Document Examination, 13*, 1–14.

Simner, M. L., Leedham, C. G., & Thomassen, A. J. W. M. (Eds.) (1996). *Handwriting and drawing research: Basic and applied issues*. Amsterdam: IOS Press.

Teuling, H. L. T., & Stelmach, G. E. (1991). Control of stroke size, peak acceleration and stroke duration in Parkinsonian handwriting. *Human Movement Science, 10*, 315–333.

Van Galen & Morasso, P. (Eds.) (1998). Neuromotor control in handwriting and drawing [Special issue]. *Acta Psychological, 100*, 1–227.

Van Gemmert, A. W. A., & Teuling, H. L. T. (Eds.) (2004). Connecting sciences using graphonomics research [Special issue]. *Motor Control, 18*, 367–566.

Varga, T., Kilchhofer, D., & Bunke, H. (2005). Template-based synthetic handwriting generation for the training of recognition systems. In *Proceedings of the 12th international conference on graphonomics society* (pp. 206–211), University of Salerno, Italy.

Wada, Y., Koike, Y., Vatikiotis-Bateson, M., & Kawato, A. (2005). Computational model for cursive handwriting based on the minimization principle. In J. D. Cowan, G. Tesauro, & J. Alspector (Eds.), *Advances in neuronal information processing systems 6 (NIPS6)* (pp. 727–737). San Francisco, CA: Morgan Kaufmann Publishers.

Wann, J., Wing, A. M., & Søvik, N. (1991). *Development of graphic skills: Research, perspectives and education implications*. London: Academic Press.

Woch, A., & Plamondon, R. (2001). Rapid movement analysis with the delta–lognormal model: Towards a better understanding of movement generation. In *Proceedings of the 10th biennial conference of the international graphonomics society* (pp. 165–169), University of Nijmegen. The Netherlands.

Woch, A., & Plamondon, R. (2003). The problem of movement primitives in the context of the kinematic theory. In *Proceedings of the 11th international conference on graphonomics society* (pp. 67–71), Scottsdale, Arizona.

Woch, A., & Plamondon, R. (2004). Using the framework of the kinematic theory for the definition of a movement primitive [Special issue]. *Motor Control*, pp. 547–557.

Zatsiorsky, V. M. (1998). *Kinematics of human motion*. Champaign, IL: Human Kinetics.