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RESEARCH ARTICLE

Extraction of delta-lognormal parameters from handwriting strokes

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Abstract In the context of the Kinematic Theory of Rapid Human Movement, handwriting strokes are considered to be primitives that reflect the intrinsic properties of the neuromuscular system of a writer as well as the basic control strategies that the writer uses to produce such strokes. The study of these strokes relies on the extraction of the different parameters that characterize a stroke velocity profile. In this paper, we present a new method for stroke parameter extraction. The algorithm is described and evaluated under various testing conditions.

Keywords Kinematic Theory, lognormal curve, neuromuscular systems, handwriting, strokes, velocity profiles, parameter estimation, non-linear regression, optimization

1 Introduction

The production of handwriting strokes is studied in many fields of computer science, particularly in pattern recognition and robotics. Indeed, many on-line handwriting recognition algorithms are based on the properties of the strokes that have been used to generate a character [1–4]. Similarly, many on-line signature verification algorithms aim at finding idiosyncratic features of strokes to characterize a signer and verify his identity [5–7]. In handwriting synthesis, strokes are used as building elements to construct character strings [8, 9], to generate training database [10], and to design interactive tools to help children learn how to write [11]. In anthropomorphic robotics, handwriting strokes are studied to explore the biomechanical principles that can be employed by humans to produce gestures and to design and control a robot arm [12]. Writing robots are also used to study ink trace depositions under controlled conditions in forensic document analysis [13].

Many of the above studies are based, directly or indirectly, upon a stroke generation model. A stroke model describes the characteristics of the pen-tip trajectory. In this perspective, a single stroke is a primitive reflecting some intrinsic properties of the neuromuscular system of a writer as well as some basic features of the control strategies that the writer uses to produce such a movement. Based on a stroke model, complex handwriting patterns such as letters and words can be considered as sequences of strokes that concatenate and superimpose one another. The parameters extracted from the strokes through the model can be used in many applications.

Among various stroke generation models, the Delta-lognormal model [14] has been found over years to be one of the most powerful models in its capability to reproduce, with a minimum error, the velocity profile of a handwritten stroke [15–17]. This model is the kernel of the Kinematic Theory of Rapid Human Movements [14, 18, 19]. In this theory, a clear operational definition of a stroke as a movement primitive is provided [20]. It describes a single stroke as a pentip trajectory with a delta-lognormal velocity profile:

$$v(t) = D_1\Lambda(t;t_0,\mu_1,\sigma_1^2) - D_2\Lambda(t;t_0,\mu_2,\sigma_2^2), \tag{1}$$

where

$$\Lambda(t;t_0,\mu,\sigma^2) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}(t-t_0)}\exp\left\{-\dfrac{1}{2\sigma^2}[\ln(t-t_0)-\mu]^2\right\}, & \text{for } t>t_0, \\ 0, & \text{else where.} \end{cases} \tag{2}$$

A stroke is thus produced by a synergy made up of an agonist and an antagonist lognormal systems. It can be described synthetically by seven parameters $(t_0, D_1, \mu_1, \sigma_1, D_2, \mu_2, \sigma_2)$ where t_0 represents the system activation time, μ_1, σ_1, μ_2, σ_2 characterize the timing properties of the agonist and antagonist neuromuscular systems in reaction to the two simultaneous input commands D_1 and D_2, respectively.

In the past decade, Guerfali and Plamondon [21,22] have developed an algorithm, hereafter referred to as INFLEX, to extract model parameters from velocity data. INFLEX takes advantage of a graphical method [23] to estimate the initial

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parameter values as the starting point and then applies a non-linear regression method [24, 25] to optimize the solution. Over the years, the performance of INFLEX has been evaluated and it has been found that the graphic method originally proposed by Wise [23] for the analysis of a single lognormal curve was not always efficient in our case where two lognormal curves have to be handled together as indicated by Eq. (1).

In this paper, we propose a new method, hereafter referred to as INITRI, to estimate the initial parameter values for the subsequent optimization by non-linear regression. We compare INITRI with INFLEX to discover their strengths and weaknesses. Then we develop a system to combine their merits and avoid their demerits.

The remainder of this paper is organized as follows. In section 2, we describe the INITRI method and the complete parameter extraction process. In section 3, we define a protocol to test INITRI and INFLEX using seven classes of ideal data. The results from the test clearly highlight the strengths and weaknesses of each algorithm and lead to the design of a system that combines the merits of both methods. In section 4, we present the testing results of all these systems under realistic noisy conditions. In section 5, we use real data sets to evaluate these systems. And finally, we present our conclusions in section 6.

2 The INITRI algorithm

The INITRI algorithm is based on an assumption that the initial rise (*initri*) of the velocity magnitude from t_0 to its first maximum is mainly contributed by the agonist neuromuscular activity. Unlike the INFLEX algorithm that uses the tangents at the inflexion points to estimate the initial parameter values, INITRI uses an analytic method instead. In this method, some points along the ascending portion of the profile are used to estimate the initial parameter values, see Fig. 1.

2.1 Estimation of the agonist parameters

Let $f(t-t_0)$ be the function of the agonist system such that:

$$f(t-t_0)=D_1\Lambda(t,t_0,\mu_1,\sigma_1^2). \tag{3}$$

Its first order derivative is

$$f'(t-t_0)$$
$$=-\frac{D_1}{\sqrt{2\pi}}\left[\frac{1}{\sigma_1(t-t_0)^2}+\frac{\ln(t-t_0)-\mu_1}{\sigma_1^3(t-t_0)^2}\right]\mathrm{e}^{-\frac{1}{2\sigma_1^2}\left[\ln(t-t_0)-\mu_1\right]^2}. \tag{4}$$

Considering the peak time and peak value of the lognormal curve, we have

$$f'(t_m-t_0)=0, \tag{5}$$

$$t_m-t_0=\mathrm{e}^{\mu_1-\sigma_1^2}, \tag{6}$$

$$f_m=f(t_m-t_0)=\frac{D_1}{\sqrt{2\pi}\sigma_1}\mathrm{e}^{-\mu_1+\frac{\sigma_1^2}{2}}. \tag{7}$$

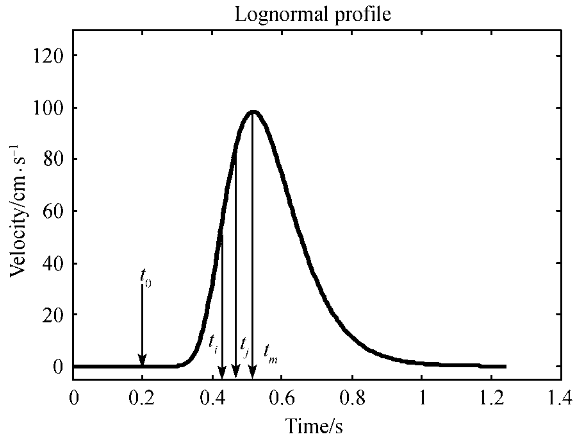

Fig. 1 Three temporal indices t_i, t_j and t_m of a lognormal curve used by the INITRI process

We assume that in [0, t_m] the superimposition effect of the antagonist system can be ignored so the values of $f(t-t_0)$ can be estimated within this region. Given a t_k, $t_0<t_k<t_m$, with a non-zero value

$$f(t_k-t_0)=\frac{1}{\sigma_i\sqrt{2\pi}(t_k-t_0)}\mathrm{e}^{-\frac{1}{2\sigma_i^2}\left[\ln(t_k-t_0)-\mu_i\right]^2}$$
$$=\frac{D_1}{\alpha_k\sqrt{2\pi}\sigma_1}\mathrm{e}^{-\mu_1+\frac{\sigma_1^2}{2}}, \tag{8}$$

where $\alpha_k>1$ is the ratio of f_m over $f(t_k-t_0)$, it follows that

$$t_k-t_0=\mathrm{e}^{\mu_1-\sigma_1^2\pm\sigma_1\sqrt{2\ln\alpha_k}}. \tag{9}$$

Because $t_0<t_k<t_m$, from Eq. (6), we have

$$t_m-t_k=\mathrm{e}^{\mu_1-\sigma_1^2}(1-\mathrm{e}^{-\sigma_1\sqrt{2\ln\alpha_k}}). \tag{10}$$

Now consider two different time occurrences t_i and t_j such that $t_0<t_i<t_j<t_m$, we have

$$\begin{cases} t_m-t_i=\mathrm{e}^{\mu_1-\sigma_1^2}(1-\mathrm{e}^{-\sigma_1\sqrt{2\ln\alpha_i}}), \\ t_m-t_j=\mathrm{e}^{\mu_1-\sigma_1^2}(1-\mathrm{e}^{-\sigma_1\sqrt{2\ln\alpha_j}}). \end{cases} \tag{11}$$

Thus, it follows that

$$\frac{t_m-t_i}{t_m-t_j}=\frac{1-\mathrm{e}^{-\sigma_1\sqrt{2\ln\alpha_i}}}{1-\mathrm{e}^{-\sigma_1\sqrt{2\ln\alpha_j}}}. \tag{12}$$

In the above equations, the velocity magnitudes at time occurrences t_i, t_j and t_m are already known, α_i and α_j are the ratio of the peak value over the values at time t_i and t_j, respectively. Thus, σ_1 can be solved without difficulty. Once σ_1 has been solved, μ_1 can be solved by Eq. (10) or (11), then D_1 by Eq. (7) and t_0 by Eq. (6), respectively.

2.2 Estimation of the antagonist parameters

Let $g(t-t_0)$ be the function of the antagonist system such that

$$g(t-t_0)=D_2\Lambda(t;t_0,\mu_2,\sigma_2^2). \quad (13)$$

$g(t-t_0)$ can be uncovered from the synthetic signal $v(t)$ once the agonist system $f(t-t_0)$ has been solved:

$$g(t-t_0)=f(t-t_0)-v(t). \quad (14)$$

Similar to the case of the agonist system, the peak time and the peak value of the antagonist system satisfy the following equations:

$$t_m-t_0=e^{\mu_2-\sigma_2^2}, \quad (15)$$

$$g_m=g(t_m-t_0)=\frac{D_2}{\sqrt{2\pi}\sigma_2}e^{-\mu_2+\frac{\sigma_2^2}{2}}. \quad (16)$$

Notice that

$$L=\int_{t_0}^{\infty}v(t)dt=D_1-D_2. \quad (17)$$

where L is the trajectory distance. Because t_0, D_1 and L are already known, we have

$$D_2=D_1-L. \quad (18)$$

Now the activation time t_0, the peak time t_m, the peak value g_m, and the command amplitude D_2 of $g(t-t_0)$ are already known, we have

$$\ln\left(t_m-t_0\right)=\mu_2-\sigma_2^2, \quad (19)$$

$$\ln\left(g_m\sqrt{2\pi}\frac{\sigma_2}{D_2}\right)=-\mu_2+\frac{\sigma_2^2}{2}. \quad (20)$$

From Eqs. (19) and (20), we have

$$\ln\left[(t_m-t_0)g_m\sqrt{2\pi}\frac{\sigma_2}{D_2}\right]=-\frac{\sigma_2^2}{2} \quad (21)$$

and hence

$$2\pi(t_m-t_0)^2g_m^2\sigma_2^2=D_2^2e^{-\sigma_2^2}. \quad (22)$$

Eqs. (22) and (19) give a complete solution to σ_2 and μ_2.

2.3 Optimization process

The initial parameter values estimated by the INITRI method may represent only a coarse solution depending on the experimental conditions under which handwriting data are collected. Starting from the coarse solution, we can optimize the parameters to get a better solution. Here we use the non-linear regression technique to minimize the distance between experimental data and the predictive model in terms of the Mean Square Errors (MSE) [25].

The complete parameter extraction algorithm thus consists of two modules: an initialization process that estimates the initial parameter values, and an optimization process that starts from the initial values and converges to an optimal solution. The complete system works as follows: given a velocity profile $v(t)$, the first process evaluates the agonist lognormal parameters as described in section 2.1. The antagonist lognormal curve is obtained by subtracting the agonist lognormal curve from the synthetic curve, and its parameters are estimated as described in section 2.2. Then the seven estimated parameter values are used as the initial conditions by the optimization module [24,25]. Both the optimal values of the seven parameters and the MSE between the original and the reconstructed velocity profiles are obtained after the optimization process converges.

3 Testing under ideal conditions

Firstly, we used ideal data to evaluate the performance of INITRI and compare it with INFLEX. For this purpose, we created seven data sets using a random generator, each set containing 1000 delta-lognormal velocity profiles with all seven parameters being randomly selected within a realistic interval [26,27]. Notice that a delta-lognormal velocity profile described by Eq. (1) may have up to two zero crossings, leading to 1, 2 or 3 peaks, we grouped these simulated curves into seven classes according to (a) the number of peaks appeared in the velocity profile, and (b) the dominant position of the antagonist component with respect to the agonist one. Figure 2 depicts a typical examples for each class C_{uw}, where the subscript u can be b (before), a (after) or s (simultaneous) and the subscript w can be 0,1,2 or i, which represents the number of zero crossings, and w = *i* (imaginary) means that there is no real roots to the zero crossing equation [18]. As one can see in these plots, the different timing of the antagonist curve versus the agonist curve generates different velocity profile patterns.

In this experiment, each random delta-lognormal curve was sampled at a rate of 200Hz to simulate the data collected from a digitizer; the discrete data then were used for the test. The seven parameters extracted by INFLEX and INITRI were compared with the true parameters and the results are summarized in Table 1. In this test, the parameters extracted by a system were considered as matching the original ones if the SNR between the original and the reconstructed curve was greater than 100 dB.

As one can see from Table 1, for the C_a classes, INFLEX performs better than INITRI, except for C_{a2}, where INITRI gets perfect results. Both algorithms have serious problems with the C_b classes, where the antagonist activity is dominant before the agonist activity. If we remove these two classes from our database, we can see, in the penultimate column, that there is a 6% difference between the two algorithms. This difference drops to 2% (last column) when all the specimens are taken into account. A vertical comparison of the results in Table 1 also emphasizes the complementarity of the two algorithms. This is mainly due to the fact that INFLEX encounters some difficulties when the main peak of the velocity profile is almost symmetric, while INITRI has some troubles with these profiles that are very asymmetric.

In an attempt to improve the performance, we have thus designed a combined system, INFLEX+INITRI that integrates both algorithms in parallel. The test results of INFLEX+INITRI under the same conditions are presented in Table 1. As one can see from Table 1, INFLEX+INITRI has a better performance than both INFLEX and INITRI. It can recover 99.07% of the original parameters for the C_a

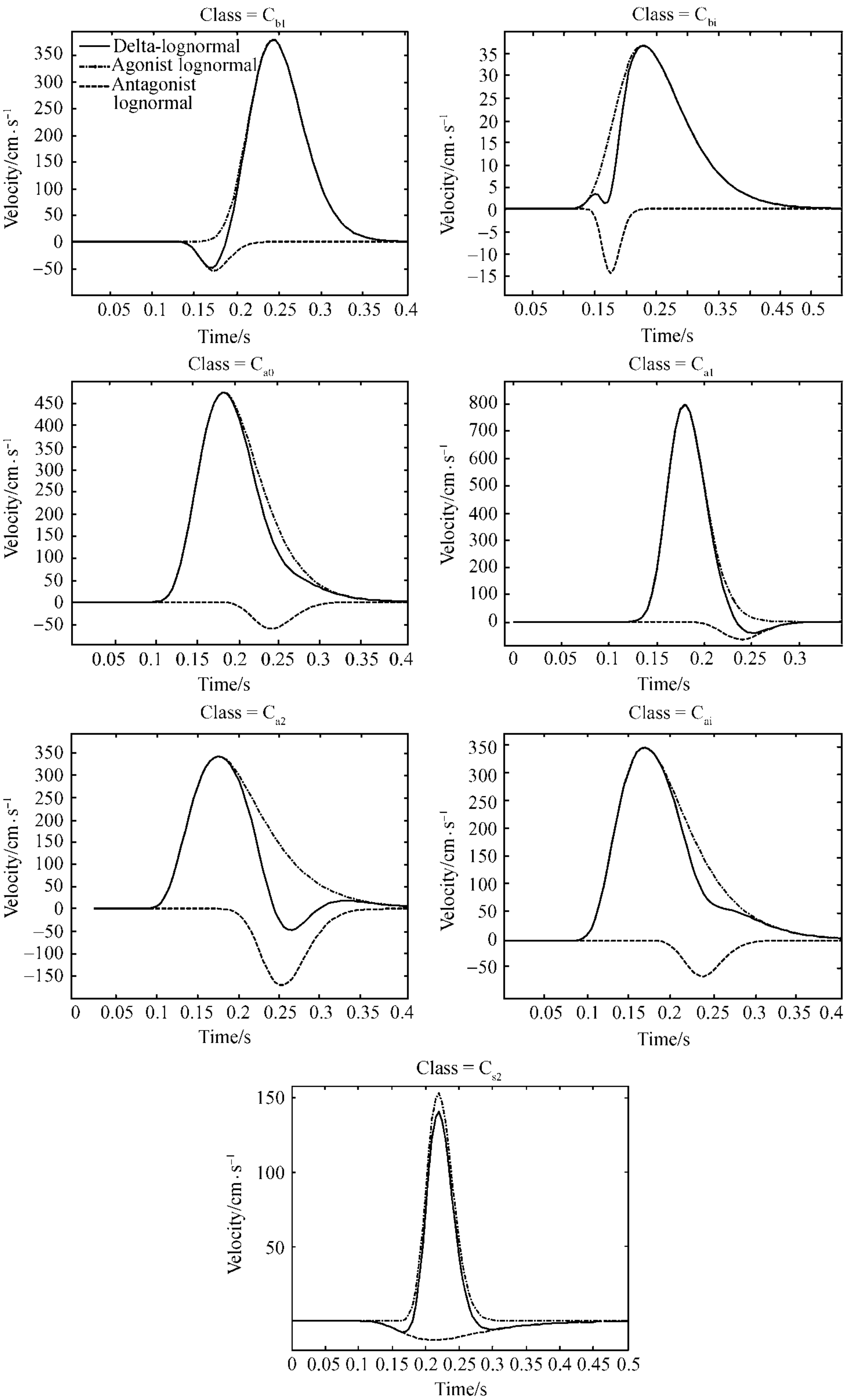

Fig. 2 Seven classes of delta-lognormal velocity profiles (solid lines) with corresponding agonist and antagonist components (dotted lines)

Table 1 Results of the tests under ideal testing conditions (Performance criterion: SNR⩾100 dB)(%)

	C_{bi}	C_{b1}	C_{a0}	C_{ai}	C_{a1}	C_{a2}	C_{s2}	Downstream only	All
INFLEX	0	9	90.1	94.6	91.9	94.6	92.8	92.8	66.41
INITRI	3	37.1	74.7	92.1	80.7	100	65.8	86.87	64.38
INFLEX+INITRI	3	41	98.5	99.4	98.4	100	94.5	99.07	76.01

classes even though it still faces problems when the antagonist activity precedes the agonist activity.

4 Testing under noisy conditions

In a real situation, the handwriting signals acquired from a digitizer tablet usually contain noise. To test the performance of our systems under noisy conditions, a 25dB noise with zero mean was added to the previous ideal data sets using a Gaussian random noise generator [27, 28]. The algorithms were applied to the noisy data to extract the seven parameters and then reconstruct the delta-lognormal curves using the extracted parameters. The algorithms were evaluated based on the mean square error (MSE) and signal-to-noise ratio (SNR) of the reconstruction. Figure 3 shows a pair of noisy profiles corresponding to two of those profiles presented in Fig. 2.

Under the noisy conditions, the extracted parameter vector normally converges towards a certain point with a non-zero MSE, and the SNR is small when the discrepancy between the noisy and reconstructed curves is large. The experimental results on the 7000 noisy curves are summarized in Table 2, where a parameter vector was assumed to have converged if the SNR between the original curve and its reconstructed version was greater than 10 dB.

As one can see from Table 2, INITRI performed better than INFLEX under noisy conditions, and the combined system (INFLEX+INITRI) converged in more than 93% of the cases.

Since we know the real parameter values of each noisy curve in this simulation experiment, we can evaluate a system regarding its capability to recover the real parameter values. Figure 4 highlights this point using the combined system. In Fig. 4 there are seven parameter plots that link the extracted parameters with the real parameters of C_{a2} data classes**.** Each plot in Fig. 4 relates to a parameter and each dot in a plot represents the best extraction result obtained by INFLEX+INITRI method. For an ideal system, all dots in a plot should be located on the 45^o oblique line. As one can see from these plots, the combined system converges on the 45^o oblique line very well regarding parameters t_0, D_1 and D_2, but it disperses from the line a little bit regarding parameters μ_1, σ_1, μ_2 and σ_2.

5 Testing with real data

To evaluate the performance of our new parameter extraction method in real applications, we conducted another experiment using real stroke data and the results are presented in Fig. 5. Figure 5(a) and (b) depict two strokes produced by a human subject on a digitizing tablet. In this experiment, the synthetic velocity profile from the two component velocities $x(t)$ and $y(t)$ of each stroke was computed using a derivative filter. The data then was input to our parameter extraction system. The results of this analysis-by-synthesis process are shown in Fig. 5(c) and (d). In each case, as one can see, the velocity profile reconstructed from the seven extracted parameters is very close to the original profile. The MSE−SNR of the two reconstructed strokes were 0.58 cm^2/s^2-29.12 dB and 0.04 cm^2/s^2-39.87 dB**,** respectively.

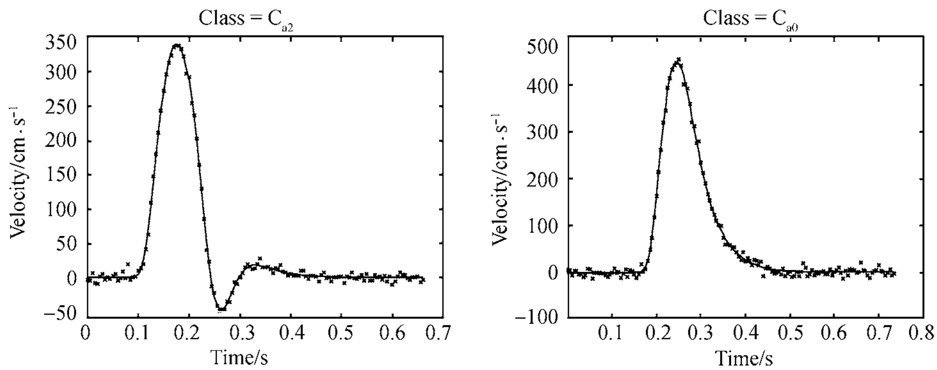

Fig. 3 Two noisy delta-lognormal profiles that correspond to two of the ideal profiles depicted in Fig. 2 (classes C_{a2} and C_{a0}, respectively), upon which 25dB noise has been superimposed

Table 2 Results of the tests under noisy testing conditions (Convergence criterion: SNR⩾10 dB)

Algorithms	Noisy data				
	% of convergence (from 7000 curves)	MSE_{mean} /cm^2·s^{-2}	MSE_{std} /cm^2·s^{-2}	SNR_{mean} /dB	SNR_{std} /dB
INFLEX	63.24	32.90	62.44	26.76	2.13
INITRI	82.90	31.98	60.91	26.62	2.28
INFLEX+INTRI	93.81	29.70	63.20	26.83	2.26

6 Conclusion

In this paper we have proposed a new method (INITRI) to estimate the initial values of the seven delta-lognormal parameters, which are used as a start point by a non-linear regression process for parameter optimization. We have compared INITRI with INFLEX (a previous method) under ideal and noisy conditions. Our experiments have shown that INITRI performed better than INFLEX under noisy conditions although the reverse was observed under ideal conditions. Since the two algorithms seemed to be complementary, we developed a combined system, INFLEX + INITRI, which inherits the merits of both methods. Furthermore, the performance of the combined system on real stroke data has been highlighted. Our algorithms are useful in various studies that focus on the intrinsic properties of strokes as building blocks for the automatic processing of handwriting in various fields of computer science [29, 30].

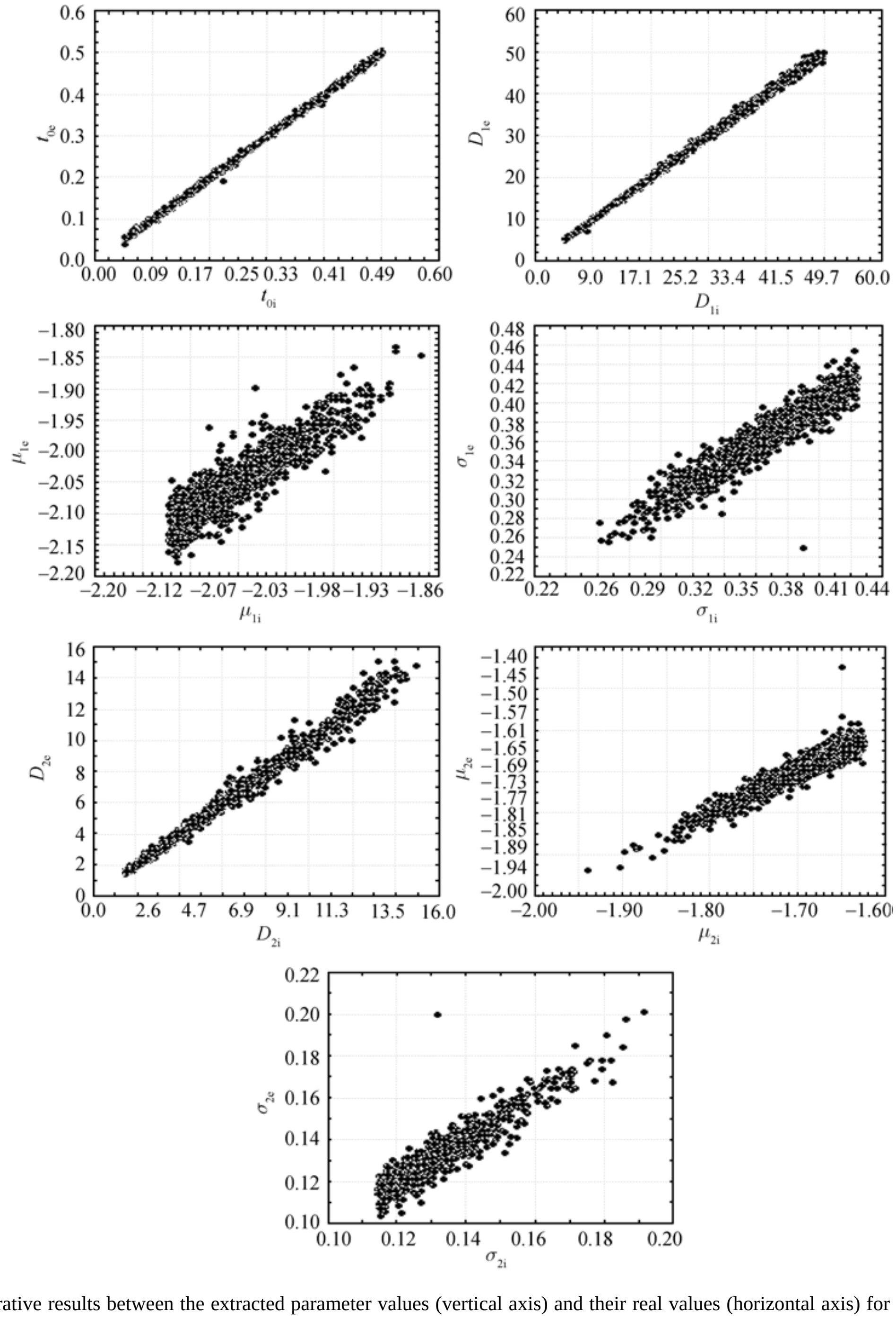

Fig. 4 Comparative results between the extracted parameter values (vertical axis) and their real values (horizontal axis) for the C_{a2} classes under noisy conditions using INFLEX+INITRI method (SNR = 25 dB). The global convergence rate was 95%

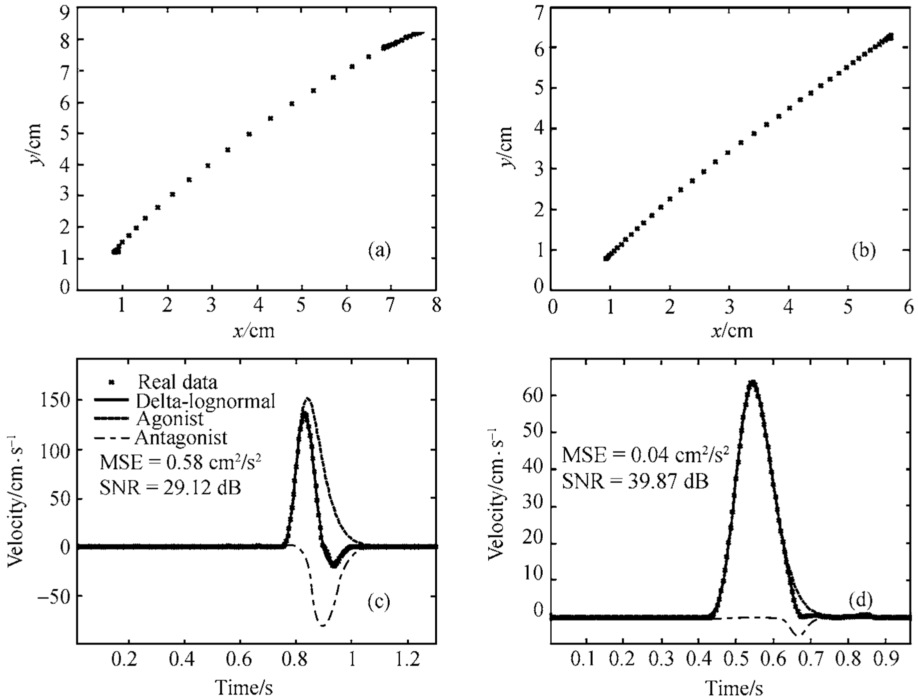

Fig. 5 System performance on real data. (a) and (b): stroke trajectory; (c) and (d): velocity profiles (cross: original profile; solid line: reconstructed profile; truncated line: agonist component; dotted line: antagonist component). Extracted parameters for (a) and (b): $t_0 = 0.704$ s, $D_1 = 17.54$ cm, $\mu_1 = -1.89$, $\sigma_1 = 0.32$, $D_2 = 8.38$ cm, $\mu_2 = -1.61$, $\sigma_2 = 0.21$, $t_0 = 0.251$ s, $D_1 = 7.63$ cm, $\mu_1 = -1.20$, $\sigma_1 = 0.16$, $D_2 = 0.26$ cm, $\mu_2 = -0.87$, $\sigma_2 = 0.05$.

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