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Can non-linear muscle dynamics explain the smoothness of handwriting movements?

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Abstract

The paper addresses the issue of the role of non-linear muscle dynamics in determining the smoothness and invariance of handwriting trajectories. In particular, a specific neuromuscular control model is described that has recently been shown to explain the detailed time course of hand stiffness during arm reaching movements. In the paper the model is applied to more complex handwriting trajectories, with the purpose of verifying to which extent the load compensation capabilities of the periphery can subserve motor equivalence. Simulations show the power of the mechanism for movements of "normal" speed. For quicker movements periphery alone is not enough and a central load compensation action is clearly required. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The smoothness of hand trajectories, in general, and of cursive handwriting, in particular, has long been the subject of study and controversy. Is it embedded into

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the motor planning processes or is it a consequence of the dynamics of the neuromuscular system? Are the motor plans analogic/smooth or symbolic/discrete? Human movements are indeed smooth, in the sense that can be described by mathematical models that maximize some kind of smoothness criterion (Flash and Hogan, 1985; Uno et al., 1989). On the other hand, the detailed analysis of the speed profile clearly shows that complex movements are segmented into sub-movements and it is quite tempting to assume that the global observed movement is just an emergent property, determined by the superposition of individually smooth primitives. In fact, an argument against a dynamic explanation is that it is difficult to reconcile it with the observed scale and shift invariance of handwriting patterns: the dynamic interaction forces in the arm are strongly non-linear and their effects are configuration-dependent, inducing patterns of anisotropy and deformation that are not invariant with scaling and shifting.

At the same time, a consistent body of knowledge is being accumulated (Gielen and Houk, 1987; Lin and Rymer, 1997; Gribble et al., 1998) that emphasizes the "surprising" effects of the non-linear dynamics of the "motor servo" (the muscular system and the associate segmental reflex mechanisms). In fact, most neuromotor models are based on an explicit/implicit linearity hypothesis that assigns a constant stiffness to the muscles, thus posing an unsolvable dilemma: if stiffness is "small" and compatible with physiologic values near equilibrium, then model-driven trajectories tend to be highly distorted; if stiffness is big enough to counteract the distorting effects of dynamics, then its value turns out to be beyond the normal physiologic range. On the contrary, the non-linear muscle model used in the paper can fit both requirements: small "real" stiffness at rest and high "virtual" stiffness during the acceleration and deceleration phases, in agreement with the experimental results of Gomi and Kawato (1996). We show that in this way the system appears to be, at the same time, highly compliant and rather insensitive to size and time scaling because it allows a satisfactory partial compensation of the dynamic interactions.

In the paper, we investigate to which extent such non-linear muscle dynamics may explain, at the same time, the smoothness and size/shift invariance of complex trajectories typical of handwriting patterns.

2. Arm and muscle model

We simulated handwriting movements in the horizontal plane by means of a planar model of the arm with two degrees of freedom (elbow and shoulder rotation, respectively) and six muscles: four single-joint muscles (pectoralis, deltoid, long head of the biceps, lateral head of the triceps) and two double-joint muscles (short head of the biceps and long head of the triceps). Moment arms of the muscles were assumed to be constant. The model has been used for the investigation of reaching movements (Gribble et al., 1998).

The muscle part of the model, illustrated in Fig. 1, is a simplified, lumped-parameter model that does not reach the level of motor units but still attempts to be

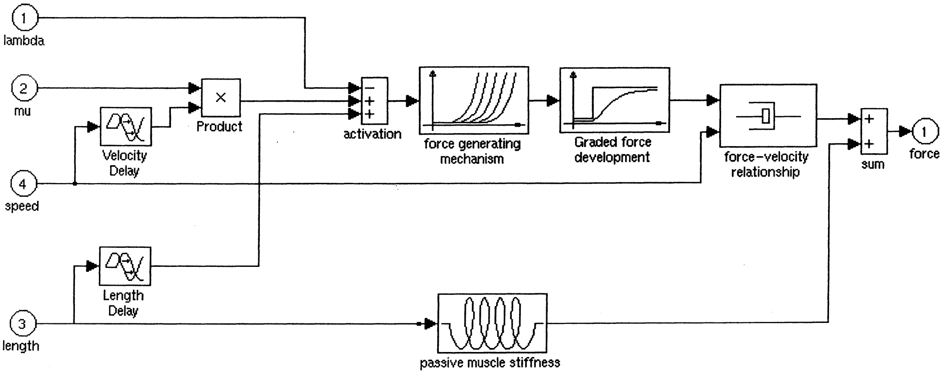

Fig. 1. Simulation model of the muscles. The model has four inputs (the command variable lambda, the parameters, the length of the muscle and its speed of contraction) and one output (the force).

biologically realistic from the point of view of system dynamics. Its plausibility is based onto two different lines of evidence: structural/local and behavioral/global. From the structural point of view, the model attempts to incorporate, separately for each muscle, a number of non-linear effects characterized by experimentally evaluated parameters. From the behavioral point of view, we checked the consistency of a multiple-muscle system, constructed with such muscular components, with experimental data about end-point impedance, that depend on the global interaction of whole musculo-skeletal system. As mentioned above, this level of analysis has been demonstrated to be sufficient to explain the characteristic time-course of arm stiffness in reaching movements (Gribble et al., 1998), thus motivating our study of the smoothness/invariance features of handwriting movements.

The main features of the model can be summarized as follows:

1. Muscle force is decomposed into a non-controllable passive component and a controllable active component:

$$f_{\mathrm{m}} = f_{\mathrm{p}} + f_{\mathrm{a}}. \tag{1}$$

2. The passive component is modeled for simplicity as a linear spring. [1]
3. The active component is modeled as the cascade of three mechanisms: *(i)* a controllable *force generation system*, which depends of the muscle activation $A(t)$ and is compatible with the family of λ-models (Feldman and Levin, 1995); *(ii)* a mechanism of *graded force development*, which takes into account the dynamics of fused tetanus formation; *(iii)* a non-linear *force–velocity relationship* which is related to the well known Hill's law.
4. The force generation mechanism is modeled as follows:

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[1] The rest-lengths were chosen as the lengths of the muscles for a reference arm posture (shoulder angle: 45°; elbow angle: 90°). The stiffness coefficients were assumed to depend linearly upon the cross-sectional areas of the muscles: 36.5 N/m (short head of the biceps), 190.0 N/m (long head of the biceps), 258.5 N/m (deltoid and pectoralis), 209.0 N/m (lateral head of the triceps), 116.3 N/m (long head of the triceps).

$$f = \rho(e^{cA} - 1), \tag{2}$$

where c is a universal parameter that characterizes the "muscle tissue" and is the same for all the muscles, whereas ρ is specific for each muscle and is assumed to depend linearly on the physiological cross-sectional area. [2] The exponential form of the function incorporates the *size principle*, which is known to characterize the recruitment of motor units during the graded build-up of muscle force. The muscle activation variable $A(t)$ is a function of muscle length l, its rate of change $\mathrm{d}l/\mathrm{d}t$, and the controllable neural input λ:

$$A(t) = [l(t-d) - \lambda(t) + \mu \mathrm{d}l(t-d)/\mathrm{d}t]^{+}, \tag{3}$$

where $[\,]^{+}$ is a ramp function (it clips the output to 0 for negative inputs), d is the delay of the segmental tonic and phasic reflexes, [3] and μ is the gain of the phasic segmental reflex; [4] λ is the centrally specified rest-length of a muscle.

5. The mechanism of graded force development is approximated, for simplicity, by means of a low-pass filter, characterized by a time constant of 15 ms (it yields a "tetanic fusion" of about 60 ms.).
6. The force–velocity relationship is approximated by a sigmoid, in agreement with the experimental data of Joyce and Rack (1969).

The dynamic model of the whole arm translates the muscle force vector f_m into the corresponding joint torque vector τ_m according to the moment-arm matrix J_m: $\tau = (J_m)^T f_m$. This vector is balanced with the internal load (due to inertial torques) and the external load (due to friction of the of the pen with the writing surface, disturbances, etc.). The combined equation can then be written as follows:

$$I(\boldsymbol{q})\mathrm{d}^2\boldsymbol{q}/\mathrm{d}t^2 + C(\boldsymbol{q}, \mathrm{d}\boldsymbol{q}/\mathrm{d}t)\mathrm{d}\boldsymbol{q}/\mathrm{d}t + (J_s)^T\boldsymbol{f}_{\mathrm{load}} = (J_m)^T\boldsymbol{f}_m, \tag{4}$$

I is the inertial matrix, C the Coriolis matrix, $\boldsymbol{q}$ is the vector of joint angles; J_s is the "spatial" Jacobian matrix that maps end-effector forces into joint torques and is a function of $\boldsymbol{q}$; J_m is the "muscle" Jacobian matrix and is constant in our case for the simplifying assumption about the moment arms of the muscles. The mechanical parameters of the arm, which determine the values of I, C, J_s, J_m, are compatible with typical human data (Winters and Woo, 1990). In the simulations we assumed that the pen friction can be neglected.

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[2] According to empirical force-length data (Feldman and Orlovsky, 1972), c can be fitted with the value 0.112 mm^{-1}. In agreement with Winters and Woo (1990), the following estimates of cross-sectional areas were used: short head of the bicepts 2.1 cm^2; long head of the biceps 11 cm^2; deltoid and pectoralis 14.9 cm^2; lateral head of the triceps 12.1 cm^2; long head of the triceps 6.7 cm^2. The associated conversion factor has been chosen to be 25 N/cm^2 in order to fit empirical measurements of arm stiffness (Tsuji et al., 1995)

[3] The estimate of d is 25 ms, in agreement with the unloading response of human arm muscles (Houk and Rymer, 1981).

[4] The estimate of μ is 0.06 s, in order to fit measured values of joint viscosity (Tsuji et al., 1995).

2.1. Motor commands

According to the structure of λ-models, the motor command vector $\boldsymbol{\lambda}(t)$ has two components: a reciprocal component $\boldsymbol{\lambda}_R$ and a co-contraction or co-activation component $\boldsymbol{\lambda}_C$. The reciprocal command corresponds to the ideal or equilibrium trajectory, expressed in muscle coordinates. If this command is produced with very small speed and acceleration, then the left-hand side of Eq. (4) is almost null, i.e. the system "slides" on the ideal path while remaining close to equilibrium: $\boldsymbol{f}_m \approx 0$ and thus $\boldsymbol{\lambda}_R \approx \boldsymbol{l}_{eq}$ (according to Eqs. (2) and (3)). This vector (6-dimensional in our case) is completely specified by the vector of free variables (2-dimensional in our case). Reciprocal commands of antagonistic muscles will vary over time in opposite directions whereas the commands of synergistic muscles will tend to co-vary.

The co-activation component, on the contrary, is a command with the same sign for all the muscles, agonists and antagonists. (It is subtracted from λ_R in all cases.) Mathematically, it must satisfy the homogeneous equation $(J_m)^T \boldsymbol{f}_m = 0$: the motor torque vector must be null although the muscle forces are not. In algebraic terms, we can say that the force vector due to co-activation commands must span the so-called *kernel-space* of J_m [5] and since this matrix is assumed to be constant, it turns out that in our model the co-activation vector is proportional to a constant vector. The meaning of this vector is the relative "mixing" proportions of muscle forces consistent with equilibrium in different parts of the work-space. [6] In the simulations, the vector was modulated by a single parameter or scalar *co-activation command*; the value was chosen in such a way to limit the corresponding muscle forces to a level comparable with the peak load-related forces and thus it was increased for quicker movements.

Let us now suppose that an equilibrium trajectory has been chosen, including its shape and timing: from this, the reciprocal time-varying command vector is directly obtained and the chosen co-contraction vector is then subtracted, thus producing the total command level of $\boldsymbol{\lambda}(t)$. The real trajectory can be computed by integrating Eq. (4) and it will necessarily differ from the virtual trajectory because it is such a difference that is the causal element for the generation of the muscle forces required for counteracting the internal and external loads. For the purpose of this paper, the equilibrium trajectory was chosen on purpose to be non-smooth, in order to test to which extent muscle dynamics might carry out, at the same time, a smoothing and load compensation action.

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[5] This is a geometric statement, independent of the muscle model.

[6] It should be noted that although the co-activation force vector $\boldsymbol{f}_m$ has a constant direction, for the assumption about the moment-arms of the muscles, the corresponding command vector $\boldsymbol{\lambda}_C$ changes direction with different levels of co-activation level, as a consequence of the non-linearity of Eq. (2).

3. Simulation results

In the simulation experiments that involved handwriting, the equilibrium-trajectory was determined by sub-sampling a digitized handwritten trace, such as an *l*-shape, thus yielding a small number of via-points uniformly spaced in time (about 5 points per stroke). The command pattern was then constructed as a linear polygon and each side was sampled at constant speed, providing a non-smooth command structure as regards both shape and timing. Moreover, this command was scaled in space and time, in order to test the size/time scaling invariance of the control mechanism. The simulations were not meant to demonstrate that the brain constructs the motor command in this way but rather to explore the smoothness/invariance issue. For this reason, it is convenient to test the system with non-smooth command patterns and the procedure above is just a simple method to generate them.

In the simulations, with increasing values of speed we also increased the level of co-contraction, in agreement with Bennett et al. (1992), Gribble et al. (1998).

Figs. 2 and 3 illustrate an example, representative of the different simulations. Fig. 2 is related to the movement at normal speed (total duration of the input command is 2 s). The top panel shows the command pattern (thick line) and the resulting output trajectory, which is smoothed and only slightly distorted. Note that the two axes are in meters and so this particular example refers to a very large movement, with the purpose to exaggerate the dynamic effects. However, for this timing the results are very similar also for centimeter-size letters. The bottom panel shows, for the same movement, the corresponding speed profile: the three peaks, which are typical for this type of letter, are an emergent property of the model dynamics because the command variables have as many sharp transients as the via-points (15 in this case). Very similar results are obtained if the number of via points is changed. [7] Fig. 3 shows the output trajectories produced with the same command pattern but different speed (the total duration of the command is 1 s in the top panel and 0.5 s in the bottom panel): the distortion of the output trajectory increases with speed.

In general, the results can be summarized as follows:

1. In all cases, the trajectories are smooth and the general structure of the speed profile is compatible with the characteristic succession of velocity peaks (one per stroke). This is remarkable because the command pattern is non-smooth and is composed of a larger set of via-points.
2. For movements with standard speed (which corresponds to a duration of each underlying stroke of about 700–800 ms) the recorded trajectory is rather similar to the equilibrium trajectory, irrespective of the spatial scaling of the command pattern.

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[7] If sub-sampling is exaggerated, the resulting curve is distorted, but this is not dependent on the muscle model per se. It only says that the motor command must contain enough information about the desired trajectory to allow a suitable interpolation mechanism to reconstruct it. The rule of thumb is to have about five via-points per stroke.

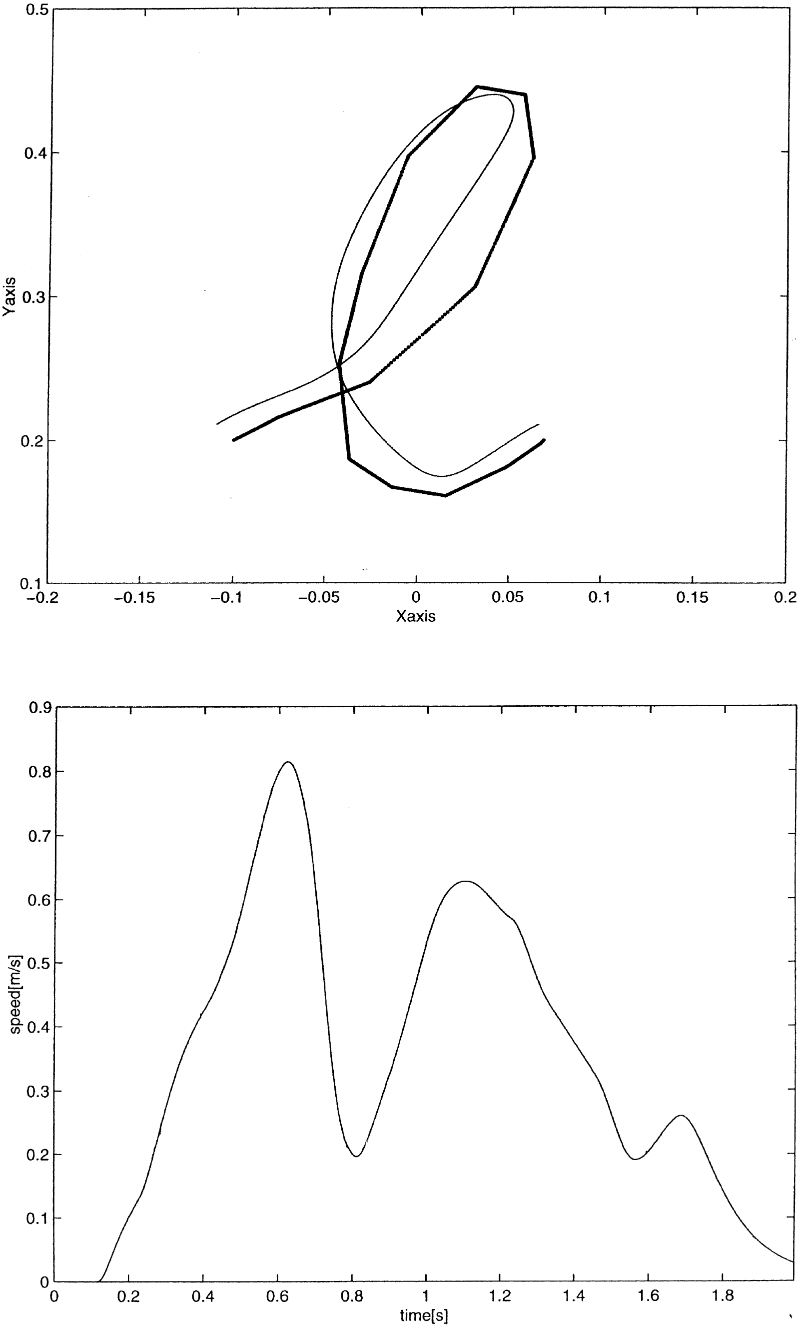

Fig. 2. Simulation of an l-shaped trajectory for normal speed (command duration 2 s). Top panel: equilibrium trajectory (thick line) and output trajectory (thin line). Bottom panel: speed profile of the output trajectory.

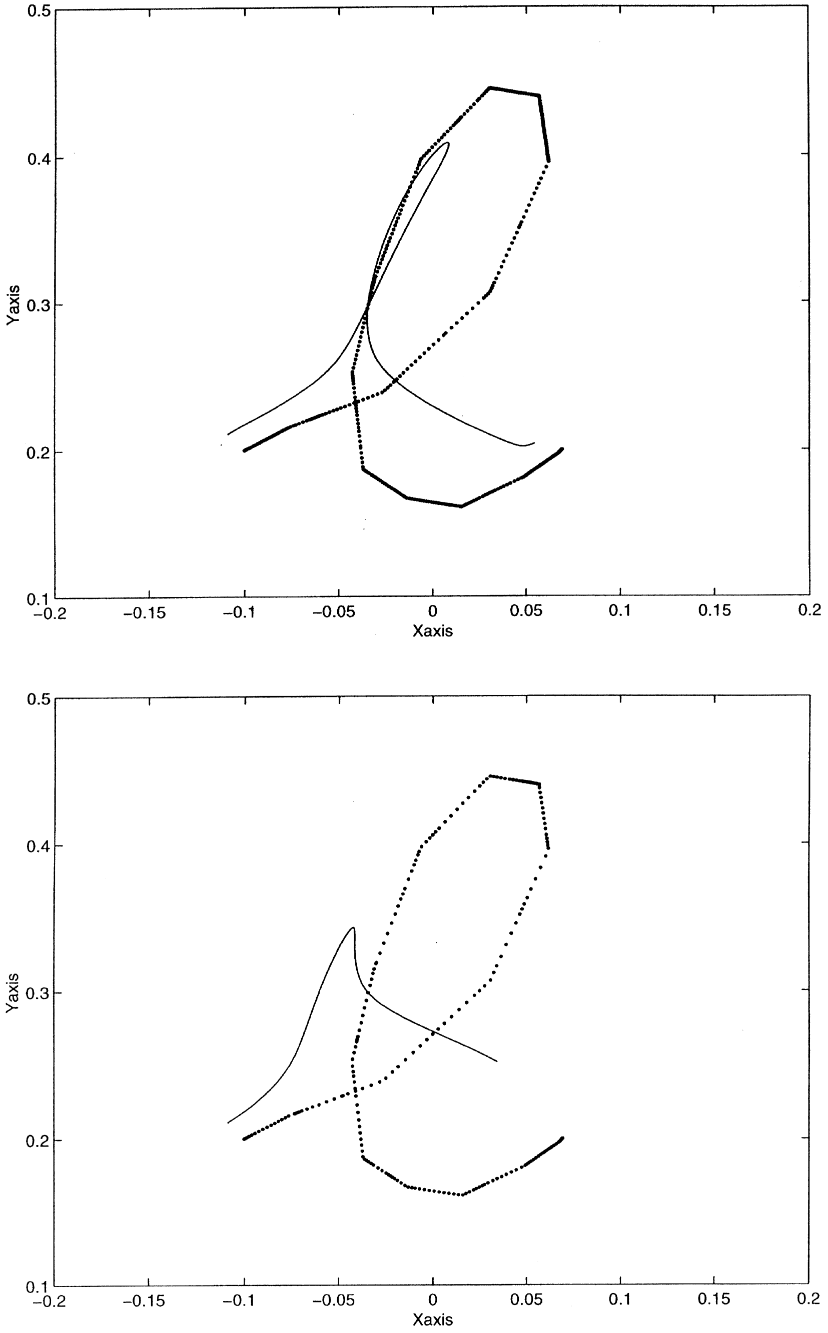

Fig. 3. Simulation of an l-shaped trajectory at high speed: equilibrium trajectory (thick line) and output trajectory (thin line). Top panel: total command duration = 1 s; bottom panel: total command duration = 0.5 s.

3. For faster movements (less than 500 ms per stroke), the discrepancy between the equilibrium and output trajectories becomes significant as well as the dependence on the spatial scaling of the command pattern.

Thus, it appears that for normal speed movements the non-linear characteristics of the neuromuscular machinery are capable to filter out the non-smoothness of the command patterns and, at the same time, to compensate the dynamic effects of the load, thus inducing the invariant spatio-temporal features. It is important to note that a linear muscle model with the same level of rest-stiffness would violate such invariance, exhibiting a much greater discrepancy between equilibrium and output trajectory even at low speed, as demonstrated by Gomi and Kawato (1996). However, the mechanism is functional up to a point: if the load is too big then the causal relationship between the motor command and the actual trajectory is lost and some additional mechanism must come into play.

4. Discussion

The simulation results are consistent with the claim (Gribble et al., 1998) that contrary to what had been suggested in other studies (Latash and Gottlieb, 1991; Gomi and Kawato, 1996), the equilibrium trajectory required for a desired handwritten trace is not a complicated non-monotonically distorted path, at least for normal movements that imply moderate amounts of dynamic disturbances. The non-linear dynamics of the muscles behaves as a non-linear filter that compensates, at least partially, the non-linear effects of the internal and external loads, thus allowing a significant degree of motor equivalence in the space domain. Thus, answering the question posed by the paper, we suggest that the observed smoothness is at least partially a direct consequence of the dynamic interactions in the muscles and associated segmental reflexes. However, the simulations also show that beyond a certain speed such a mechanism becomes insufficient, suggesting that the timing structure of the recorded movements cannot be determined solely by periphery but must somehow be an intrinsic property of the central command patterns. In any case, we are not suggesting that for most normal movements, involvement of the central nervous system is not necessary. We simply observe that the system can ignore internal and external loads most of the time, while focusing its computational power on the basic spatio-temporal sketch of the desired trajectories: the muscle properties are good enough to smooth out the command patterns and compensate most of the loads, while providing a good level of invariance.

All together, we can say, in contrast to what is supported by some authors, that smoothness and motor equivalence is not the result of a direct optimization process but is the emergent property of the interplay between peripheral features (the filtering/load-compensation properties of the neuromuscular actuators – for slow movements) and central mechanisms (the linear superposition of primitives and the extra load compensation – at high speed).

In fact, the intrinsic motor discretization of complex handwritten shapes into sequences of simple submotions is not ruled out by the simulations but, on the contrary, provides a natural ground of complementarity between peripheral and central mechanisms. The central nature of the segmentation is consistent, on the one hand, with the phasic, burst-like nature of many neurons of the motor cortex (Fetz et al., 1980) and, on the other, with the irregularity of unnaturally slow movements (Vallbo and Wessberg, 1993). Moreover, the stroke-segmentation of handwritten traces is quite compatible with the quantization of human motion that characterizes accurate reaching movements (Burdet and Milner, 1998). Such movements, in agreement with the early observations of Woodwarth (1899), are composed of two different phases, an approach and an adjustment phase, with a speed–accuracy trade-off that has been characterized in a quantitative way by Fitts (1954). The underlying discrete nature of the centrally generated commands is strongly suggested by the clearly segmented structure of arm movements in infants (von Hofsten, 1991): with ontogenetic development and learning, the blending algorithm is optimized, while preserving the same basic primitives.

As regards the load compensation at high speed, we argued elsewhere (Morasso and Sanguineti, 1997) that it is not appropriate to consider it in terms of equilibrium trajectory but as an independent mechanism probably related to the cerebellar machinery, which operates in parallel and additively with respect to the cortico-spinal machinery responsible of crafting the equilibrium trajectories.

References

Bennet, D.J., 1993. Torques generated at the human elbow joint in response to constant position errors imposed during voluntary movements. Experimental Brain Research 95, 488–498.

Burdet, E., Milner, T.E., 1998. Quantization of human motions and learning of accurate movements. Biological Cybernetics 78 (4), 307–318.

Feldman, A.G., Levin, M.F., 1995. The origin and use of positional frames of references in motor control. Behavioral and Brain Sciences 18, 723–745.

Fetz, E.E., Cheney, P.D., Mewes, K., Palmer, S., 1980. Control of forelimb muscle activity by populations of corticomotoneural and rubromotoneural cells. Progress in Brain Research 80, 437–449.

Fitts, P.M., 1954. The information capacity of the human motor system in controlling the amplitude of movement. Journal of Experimental Psychology 47, 381–391.

Flash, T., Hogan, N., 1985. The coordination of arm movements: an experimentally confirmed mathematical model. Journal of Neuroscience 7, 1688–1703.

Gielen, C.C.A.M., Houk, J.C., 1987. A model of the motor servo: incorporating nonlinear spindle receptor and muscle mechanical properties. Biological Cybernetics 57, 217–231.

Gomi, H., Kawato, M., 1996. Equilibrium-point control hypothesis examined by measured arm stiffness during multijoint movments. Science 272, 117–120.

Gribble, P.L., Ostry, D.J., 1996. Origins of the power law relation between velocity and curvature: modelling the effects of muscle mechanics and limb dynamics. Journal of Neurophysiology 76, 2853–2860.

Gribble, P.L., Ostry, D.J., Sanguineti, V., Laboissière, R., 1998. Are complex control signals required for human arm movements?. Journal of Neurophysiology 79 (3), 1409–1424.

Houk, J., Rymer, W., 1981. Neural control of muscle length and tension. In: Brooks, V.B. (Ed.), Handbook of Physiology, The Nervous System, vol. II, part 1, ch. 8. American Physiological Soc., Bethesda, MD, pp. 257–323.

Joyce, G.C., Rack, P.M.H., 1969. Isotonic lengthening and shortening movements of cat soleus muscle. Journal of Physiology London 204, 475–491.

Latash, M.L., Gottlieb, G.L., 1991. Reconstraction of joint compliant characteristics during fast and slow movements. Neuroscience 43, 697–712.

Lin, D.C., Rymer, W.Z., 1997. Nonlinear damping properties and postural stability of the neuromuscular system. IEEE EMBS, Chicago.

Morasso, P., Sanguineti, V., 1997. From cortical maps to the control of muscles. In: Morasso, P., Sanguineti, V. (Ed.), Self-Organization, Computational Maps, and Motor Control. Elsevier, Amsterdam, pp. 547–592.

Tsuji, T., Morasso, P., Goto, K., Ito, K., 1995. Human hand impedance characteristics during maintained posture. Biological Cybernetics 72, 475–485.

Uno, Y., Kawato, M., Suzuki, R., 1989. Formation and control of optimal trajectory in human multijoint arm movement: minimum torque-change model. Biological Cybernetics 61, 89–101.

Vallbo, A.B., Wessberg, J., 1993. Organization of motor output in slow finger movements in man. Journal of Physiology 469, 673–691.

von Hofsten, C., 1991. Structuring of early reaching movements: a longitudinal study. Journal of Motor Behavior 23, 280–292.

Winters, J., Woo, S.Y., 1990. Multiple Muscle Systems: Biomechanics and Movement Organization. Springer, New York.

Woodworth, R.S., 1899. The accuracy of voluntary movement. Psychological Review Monograph Supplements.