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Chapter 10

Handwriting Movement Control

Hans-Leo Teulings
Motor Control Laboratory, Arizona State University, USA

1 INTRODUCTION

Among the many motor activities - displacement of the body, maintaining posture, grasping and manipulating objects – handwriting distinguishes itself in that it is a learned and generally practiced human skill. For that reason, the motor control aspects of handwriting are both interesting and important. Being a learned skill, handwriting is related to typewriting, speech, sign language, and Morse coding. This chapter focuses on psychological, neurological, biomechanical, and computational theories of handwriting production and intends to draw parallels with these related skills, and especially with typewriting. Handwriting may have many features in common with these related motor skills so that a unified theory of these skills seems feasible. However, a unified cognitive theory of all motor skills, including grasping, posture, gait, jumping, or navigation is still lacking. This chapter intends to present the skill of handwriting, in the context of other motor skills and general motor concepts. However, handwriting skill includes only a limited domain of the basic motor skills. This may limit generalization of the knowledge of the handwriting motor system to other motor skills, but at the same time this limited domain allows simpler theories. The domain of handwriting skill is limited by the following basic features:

(1) The aim is to translate a two-dimensional graphical structure into a fixed sequence of movements. Performance is concerned mainly with the spatial structure rather than with the temporal structure.
(2) Although seemingly continuous, the handwriting movement can be considered as a sequence of discrete actions, similar to the typewriting strokes. Namely, the handwriting movement forms a discrete sequence of ballistic movement segments, or handwriting strokes, which are executed at a near maximum rate of about 10 strokes per second, just like typewriting strokes.
(3) Handwriting requires only small movement amplitudes (e.g., 0.5 cm), small flexions and extension, and small force levels so that biomechanical constraints are minimal.

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(4) The movement patterns are the result of an abstract motor program, which can be executed largely independent of visual feedback, proprioceptive feedback, friction, gravity, inertia, instructed speed, writing size, and muscles and limbs involved.

Cursive script can be recorded using a commercial digitizing tablet connected to a computer. As the position of the pen tip on the tablet is digitized at a fixed, high frequency, all kinds of time functions such as positions, velocities, accelerations in horizontal and vertical directions can be estimated (Teulings & Thomassen, 1979; Teulings & Maarse, 1984) (See Figure 1). When recording the movement of the pen tip during fluent writing the tangential (or absolute) velocity time function will show a sequence of unimodal peaks: the ballistic "strokes". A closer look reveals that the time moments of the valleys between the velocity peaks mostly correspond with time moments of high curvature (i.e., the reverse of the curve radius, or the radius of the hitting circle). It appears that after each ballistic stroke the movement direction is changing sharply to another target in the writing plane.

1.1 Recent overviews

The motor aspects of handwriting have gained attention in an international and multidisciplinary context resulting in various edited volumes (e.g., Wing, 1979; Barbe, Lucas, & Wasylyk, 1984; Thomassen, Van Galen, & De Klerk, 1985 -- in Dutch and geared towards educationalists; Kao, Van Galen, & Hoosain, 1986; Plamondon, Suen, & Simner, 1989; Plamondon & Leedham, 1990; Van Galen, Thomassen, & Wing, 1991; Wann, Wing, & Søvik, 1991). Also several monographs have been devoted to specific areas of handwriting such as biomechanical and computational modeling of trajectory formation (Maarse, 1987; Schomaker, 1991; Dooijes, 1984; Raibert, 1977; Kadirkamanathan, 1989; MacDonald, 1966), top-down handwriting movement control model (Teulings, 1988), and educational and developmental aspects (Meulenbroek, 1989; Søvik, 1975; Wann, 1988; Mojet, 1989; Blöte, 1988; Sassoon, 1988). Recently, several overviews have appeared on drawing and handwriting: An integrated handwriting model of mainly the higher levels of control (Van Galen, 1991), several intriguing topics of handwriting motor control (Rosenbaum, 1991), neurophysiological aspects for the purpose of forensic expertise (Baier & Bullinger-Baier, 1989), and signature verification (Lorette & Plamondon, 1990). The overviews, which are overlapping perhaps the most with the present one are Thomassen and Van Galen (1991), who included a brief history of the art of writing, and Thomassen & Teulings (1993), who described the handwriting movement in less technical terms. Finally, Wing (1978) deserves attention because of the overview of less known, early work on programming and biomechanical aspects of handwriting.

Experimental handwriting research as a multidisciplinary approach towards understanding of human motor control, also known as "graphonomics", is concerned with the dynamical measurement of handwriting. Handwriting research is often brought into connection with graphology, which attempts to study how to draw conclusions about personality from pages of (static) handwriting trajectories. However, the complex discipline of graphology is still remote from its empirical foundation (e.g., Wing & Watts, 1989; Tripp, Fluckiger, & Weinberg, 1957).

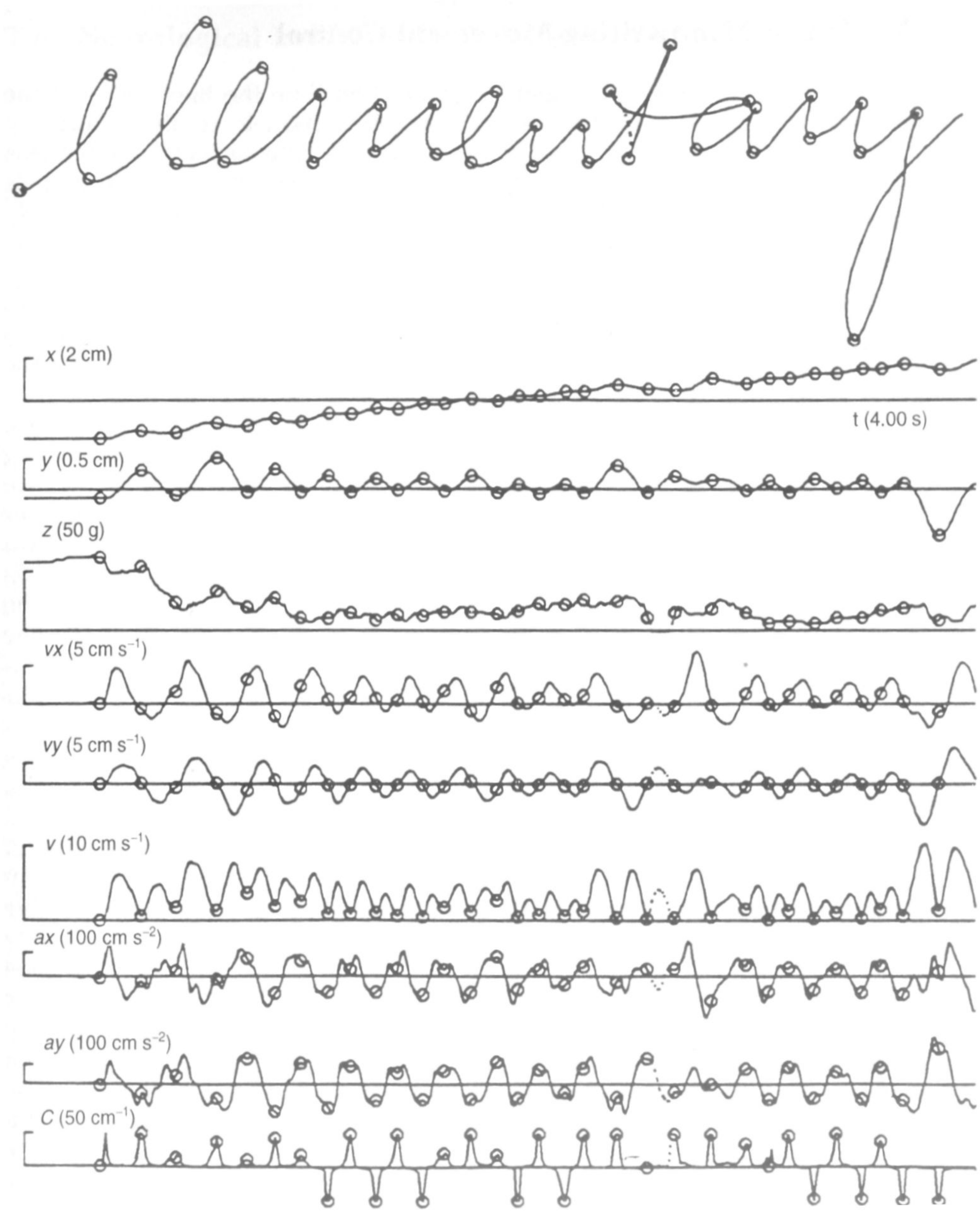

Figure 10.1: A recorded handwriting pattern with the time functions: x, horizontal coordinate; y, vertical coordinate; z, axial pen pressure; vx, horizontal velocity; vy, vertical velocity; v, absolute velocity; ax horizontal acceleration; ay, vertical acceleration; C, Curvature, i.e. the inverse of curve radius, in calibrated scales. Circles indicate the segmentation points between ballistic strokes on the basis of relative minima of v. Dotted traces refer to movements above the paper.

1.2 Models of Handwriting Movement Control

The present chapter attempts to organize for the first time the knowledge of the handwriting motor system from global to detailed models in three sections: macroscopic, microscopic, and computational models. Within each of these sections the models are arranged from high-level to low-level. The macroscopic models have several serial and parallel-working modules. A new aspect is that entirely different paradigms, such as slips of the pen, effects of neurologic disturbances, and movement latencies and durations, provide a consistent picture of the nature of these discrete modules. The microscopic models focus on specific modules of the handwriting motor system. Here all limited-scope, qualitative and quantitative models can be found. Finally, the computational models focus on the generation and simulation of handwriting movements (i.e., dynamic pen movements). Due to the limited frequency bandwidth of the handwriting apparatus, theoretically four parameters per stroke are required to describe handwriting patterns sufficiently accurate. In this chapter a first attempt is made to compare the parsimony of computational models. Most models use four parameters per stroke. If more parameters are employed the model is performing rather a curve fit. If less parameters are employed the model may be intelligent. This is interesting if a computational model intends to mimic certain aspects of the biomechanical structure of the handwriting apparatus or of the architecture of the nervous system.

2 MACROSCOPIC MODELS

The motor process of handwriting production can be divided into a number of subprocesses, which are supposed to take place in separate modules. The macroscopic models are modular models, and describe a large portion of the handwriting motor system. First the case will be considered that the subprocesses operate strictly sequentially, and subsequently the strategy that subprocesses operate in a timesharing or a (partially) parallel fashion. The modular models subsume part of the typewriting motor system, which shares some of the modules of the handwriting motor system while some other modules are analogous. Under the assumption that modules possess general principles of the motor system, research based on key pressing, tapping, and typewriting is sometimes used to support models of handwriting. The modules discerned are central modules. More peripheral effects due to nerve transmission, peripheral feedback, and muscle-force generation will not be discussed extensively. Interesting, but also not discussed are macroscopic effects of reduction of physiological stress during calligraphy and handwriting (e.g., Kao & Robinson, 1987).

Segmenting behavior into serial modules has been one of the main aims of the traditional motor research (e.g., Sanders, 1980). Also in handwriting production, evidence for separate modules has been found. Higher-level modules have been hypothesized on the basis of slips of the pen (Van Nes, 1971; 1985; Ellis, 1982), neurologic disturbances (Margolin, 1984; Caramazza, Miceli, & Villa, 1986), while lower-level modules have been hypothesized on the basis of data on movement dynamics such as delays in movement initiation or movement execution (Van Galen & Teulings, 1983; Van Galen, 1991).

2.1 Neurologic Evidence for Functional Modules

Modularity is not just a way to capture a complex system but is also based upon neurological data. Modularity may explain the great flexibility of human behavior. For example, the sequence of motor commands can be fed to any set of muscles such that writing at different sizes or with different limbs still produces similar writing patterns (Bernstein, 1947). Correlations of psychomotor features across subjects suggest single modules responsible for certain features. For example, timing speed (Keele & Hawkins, 1982), timing accuracy, and force-amplitude accuracy (Keele, Ivry, & Pokorny, 1987) seem to correlate between finger and arm. On the other hand, timing accuracy and force-amplitude accuracy do not seem to be correlated. This suggests that different modules are responsible for timing and for force control (Keele et al., 1987). Moreover, neurological evidence suggests that timing computations depend on the cerebellum while force regulation depends on the basal ganglia (Keele & Ivry, 1991).

The more or less specific disturbances caused by anatomically localized lesions in neurologic patients are also in favor of functional modules. For example, both oral and handwriting spelling showed similar frequencies of disruptions, suggesting that a hypothetical common module such as a graphemic buffer is impaired (Caramazza et al., 1986). Analogously, Boyle and Canter (1987) observed in a patient with a left-hemispheric infarct that letter-sequence errors were similar in the patient's handwriting, typewriting and speech. This confirms that a single, common, high-level module exists which underlies the errors. Furthermore, neocortical lesions may lead to more specific letter-formation errors whereas basal-ganglia disruptions (e.g., Parkinson's disease) rather lead to improper neuromuscular activity to execute adequate letter sizes (Margolin & Wing, 1983). In agreement with this, Teulings and Stelmach (1991, 1992) found evidence that Parkinsonian handwriting shows more impairment in terms of force amplitudes than durations. Dal Bianco (1944) (quoted by Denier van der Gon & Thuring, 1965), states that particular cerebellar injuries may cause movement disturbances which are more related to the horizontal, left-right movement than to the vertical, top-bottom movement. In summary, these observations suggest that specific component skills are disrupted in patients with anatomically localized impairments.

Interesting observations were made by Patterson and Wing (1989), suggesting that even storage of handwriting patterns may be modular. This conclusion was based upon a patient suffering from a left parietal stroke who showed disruption of the lower-case cursive-script allographs but relatively little disruption of the upper-case handprint allographs. The cursive-script allographs took also much more time to prepare than to execute, which indicates that it may be a retrieval problem. Furthermore, sometimes a similar cursive-script allograph was produced instead of the correct one (e.g., "a" instead of "g"). Remarkably, there were no problems in the patient's signature. There was also no problem in spelling. This suggests that well-defined parts of the motor-pattern store or of the motor-pattern retrieval were disrupted.

2.2 Overview of Handwriting and Typewriting Modules

Figure 2 provides an overview of the modules discerned in the handwriting or typewriting motor system and probably also in the motor systems describing related skills. The modules are

The letters to be written are in the
Graphemic Buffer
(containing the graphemic code, i.e. the orthographic information of a word)
↓
The buffer's contents are sent to the
Allographic Store
(containing the allographic code, i.e. the letter shapes in the context of a syllable)
↓
The retrieved code is stored into the
Allographic Buffer
↓
And sent to the
Graphic Motor-Pattern Store
(containing the graphic motor-pattern code, i.e. the stroke sequence of an allograph)
↓
the retrieved code is stored into the
Graphic Motor-Pattern Buffer
↓
at movement initiation the buffer's code is updated by the
Parameter Setting Process
(set nonmuscle specific parameters, e.g. stroke size)
↓
and the
Motor Initiation Process
(set muscle specific parameters, e.g. stroke orientation, force)
↓
Nerve transmission, activation of synapses and internal feedback yield
Muscle Contractions
↓
so that the pen tip moves, which can be analyzed after
Recording and Processing

Figure 10.2: An overview of the handwriting modules, their contents and the operation taking place between and inside them. The graphemic buffer is common for handwriting and typewriting. The lower-level modules may be analogous for typewriting.

primarily based upon Ellis (1982). His model comprises two parallel input paths to a central, cognitive system, to perceive spoken and written information and two parallel output paths, to produce speech and handwriting. Certain modules concern retrieval from a long-term memory, the storage into a short-term buffer, the ordered retrieval from a short-term buffer, or merely updating the contents of a buffer by the substitution of specific parameters.

Margolin (1984) introduced a typing branch analogous to the above handwriting branch. He suggested that the graphemic code is transformed into the allographic code, both in typing and in writing. Then, analogous to the graphic motor-pattern code, a "typing motor-pattern code" is generated, specifying the sequence of key strokes. To complete the analogy with the handwriting motor system, first the non-muscle specific, and then the muscle-specific parameters are substituted. In this section, findings on handwriting and typewriting are fitted into a single framework.

2.3 Higher-Level Modules Reflected by Slips of the Pen

Errors in speech, typewriting, and handwriting (Ellis, 1982), or, "slips of the tongue", "slips of the finger", or "slips of the pen", respectively, provide interesting possibilities to hypothesize the existence of a specific sequence of modules. The highest-level module we wish to consider here is the "graphemic buffer" (See Figure 3). This buffer contains the graphemic code needed to correctly spell the to-be-produced word. A grapheme is a letter without details as to whether it is a capital or lower case, handprint or any specific shape of cursive-script letter (See Figure 4). Therefore, the graphemic code does not yet specify the shape. In the graphemic buffer confusions between identical graphemes, but different allographs, occur (e.g., "to English" => "to eng..."). Furthermore, anticipations and perseverations (e.g., "Cognitive" => "Go...") may occur at this graphemic level where identical graphemes, e.g., "g" and "G", interact. It may be noted that the writer discovers the slip of the pen soon after it was made and stops writing so that it is unknown how the writing pattern would have been completed. Similarly, in typewriting, the inter-key period immediately after an erroneous key press is prolonged, although the typing sequence is mostly not interrupted (Salthouse, 1984).

The contents of the graphemic buffer are sent to an allographic store in order to retrieve the "allographic code" specifying the shape of each grapheme (but not the strokes' sequence). Different shapes of a grapheme are called "allographs". At this level, confusions between identical allographs may occur. The mutually affecting allographs may be separated several letter positions in a word. For example, omissions of one of the two identical allographs occurring in a word (i.e., letter masking) (e.g., "satisfactory" => "satifa..."; "listening" => "listeing") and haplographies (e.g., "dependence" => "depence"). These errors rarely occur between different allographs of the same grapheme (e.g., "Multimeter" => "Multie...") (Van Nes, 1985; Ellis, 1982), which confirms the assumption that letter masking occurs at the allographic level rather than at the graphemic level.

The retrieved allographic code is temporarily stored in the "allographic buffer". Ellis concluded that the allographic code describes the strokes of a grapheme but not their sequence or execution directions so that reversals (e.g., "p" versus "g" in "pagoda" => "padoga") and substitutions (e.g., "b" versus "p" in "Ambiguous" => "Amp...") involving similar allographs may occur. Similar allographs have similar shapes but the strokes may be performed in different orders. That these errors occur at a lower level than the previous errors may be derived from the observations that there is no context allograph facilitating letter substitution, and that the affected allographs are only few letter positions apart.

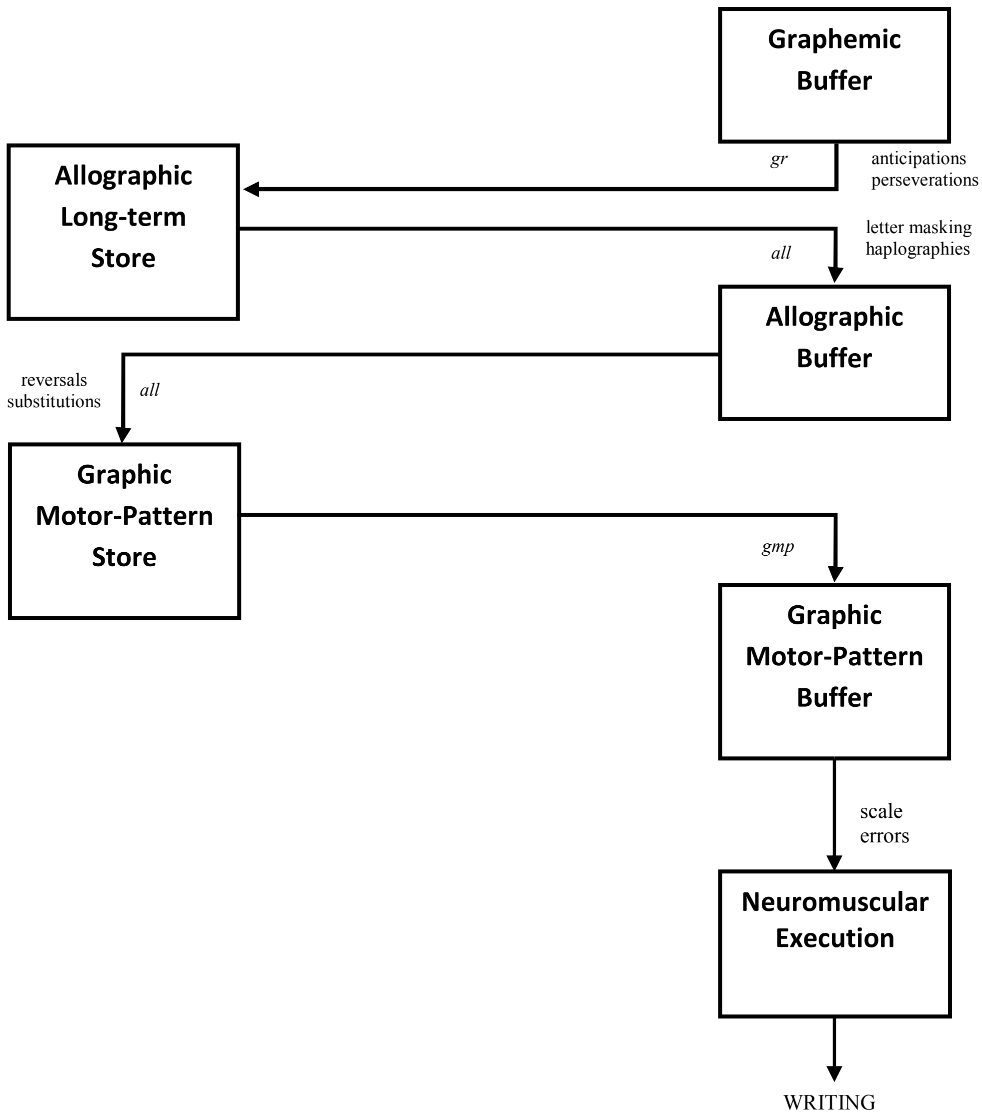

Figure 10.3: The various slips of the pen occurring at or between different modules. The neuromuscular execution still consists of two modules: the muscle-independent parameter setting and the muscle-dependent motor initiation, gr, Graphemic code; all, allographic code; gmp, graphic motor-pattern code. [From Ellis, 1982.]

The proper stroke sequence is retrieved by sending the allographic code to the "graphic motor-pattern store", yielding the graphic motor-pattern code. Extending ideas of Van Galen (1980), Ellis assumes that the graphic motor-pattern code prescribes the sequence of strokes and also the relative sizes required to perform the allograph but not the absolute size. Subsequently, the graphic motor-pattern code is stored in a graphic motor-pattern buffer, awaiting movement initiation. However, the buffered movement code still has to be completed with the current

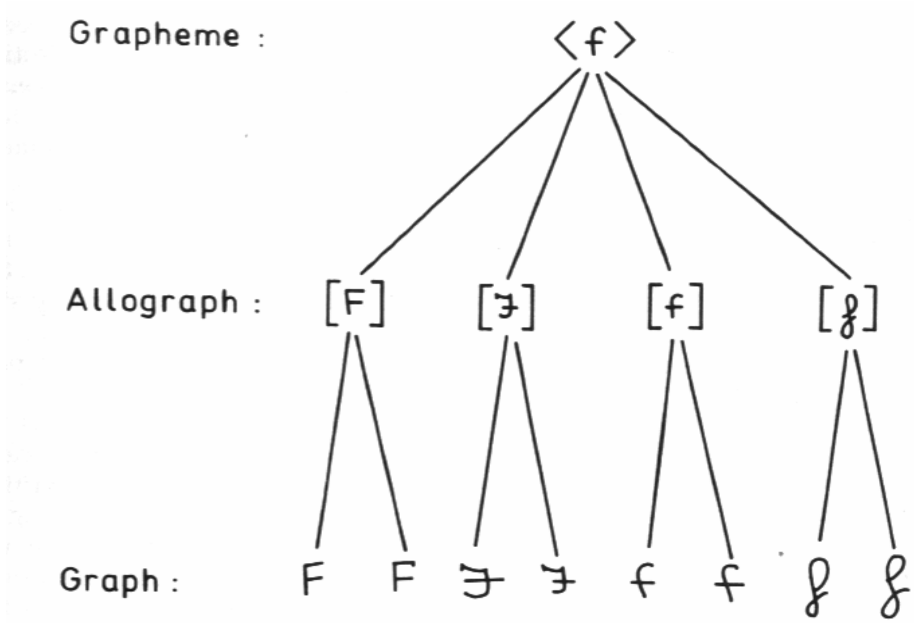

Figure 10.4: The abstract grapheme 'f' may be represented according to various allographs, Each realization of an allograph is called a graph and shows only variations due to motor noise. [From Ellis, 1982.]

scale factors like absolute sizes and starting positions. Thus, pattern errors cannot occur at this level anymore, but only scale errors. E.g., the size reduction needed to superscript a character may be omitted, but rarely the wrong character is superscripted. Finally, muscle-specific parameters have to be specified, as will be explained next.

2.4 Lower-Level Modules Reflected by Handwriting Movement Dynamics

A different type of paradigm is required to find evidence for distinct modules in the lower level of the motor system. The classical way of distinguishing modules is based on Sternberg's (1969) additive-factor method (e.g., Sanders, 1980). This method has been successful in the 80s for discovering serial modules in reaction-time paradigms. The method assumes that each module requires some hypothetical processing time, and that the reaction-time period is exactly the sum of all modules' processing times. Then, the mean reaction time equals the sum of the mean processing times. Therefore, if two experimental variables significantly affect the component times of different (sets of) modules, then the reaction-time data should be additive and not show any interaction between these variables. Even stronger predictions are possible if the component times are statistically independent. Then, not only are the means additive, but also the variances. Reversing this logical conclusion yields the speculation that if two experimental variables show significant main effects and a nonsignificant interaction, they might affect different modules. Common sense and experience of the researcher is used to choose the sequence of the module that are supposed to be affected. Using the additive-factor method, Van Galen and Teulings (1983) found indications for the existence of two lower-level modules below the previously mentioned graphic motor-pattern store where the sequence of strokes is represented as an abstract movement code.

The first lower-level module concerns "parameter setting": global, muscle-independent parameters such as position, size, speed, and force are substituted in the abstract graphic motor-pattern code. Van Galen and Teulings (1983) argued that the size of writing is not necessarily a muscle-specific parameter because writing size can be adjusted, within certain limits, without changing the roles of the muscles involved. Therefore, writing size is an experimental variable affecting the parameter setting module.

The second lower-level module concerns "motor initiation": the actual muscles and motor units are recruited. Orientation of the baseline and slant are typically muscle-specific parameters. To argue this, the authors hypothesized that writing movements are organized in terms of two muscle systems, one corresponding to finger-joint movements and one to wrist-joint movements. Each muscle system is responsible for movements in different orientations. Therefore, changing the orientation of the baseline or the slant (i.e., slope of the downstrokes relative to the baseline, Maarse & Thomassen, 1983) would imply that the roles of these muscle systems change. So orientation of the writing pattern is an experimental variable affecting the latter module.

The data in Van Galen and Teulings (1983, Fig. 3) show significant main effects of reaction time as a function of size and orientation but a nonsignificant interaction. For example, reaction time, defined as the latency between imperative stimulus and writing initiation, was for the normal orientation and the small letter width (i.e., 0.33 cm) 651 ms and for the large letter width (i.e., 1 cm) 667 ms (yielding a difference of 16 ms), and in the vertical-downward-orientation condition 710 and 728 ms, respectively (yielding a similar difference of 14 ms, thus rejecting any interaction). These data support a conclusion that two experimental variables exist which may affect different lower-level modules: size, probably affects the parameter-setting module, and orientation probably affects the motor-initiation module.

What could be the sense of so many modules after generating the graphic motor-pattern buffer? Van Galen (1980) suggested that the last-minute substitution of global and muscle-specific parameters is more efficient than storing and retrieving them, as many of the lower-level parameters are frequently changing between instances of handwriting production. For example, while writing a line on a page from left to right, the orientation of the hand changes gradually. However, the effectors involved, seem to compensate for the varying arm orientations, so that the orientation and slant of the writing pattern vary remarkably little across the line (Maarse, Schomaker & Thomassen, 1986). Therefore, it would not be efficient to store orientation or slant parameters in the graphic motor pattern. Indeed, subjects can easily change the orientation and slant of their writing, either voluntarily or induced by distorted feedback (Pick & Teulings, 1983). Furthermore, this principle of substitution of muscle-specific parameters provides an explanation of the "motor equivalence" phenomenon: the shape of a movement pattern is only marginally dependent upon the muscles that execute the final movement (Bernstein, 1967). There is a tradeoff between completely stored movements, allowing quick movement preparation, and abstract movement patterns with several processing modules, allowing great flexibility.

2.5 Higher-Level Modules Reflected by Typewriting Errors

Also in typewriting, higher-level and lower-level modules can be discerned on the basis of typing errors. Rumelhart and Norman (1982) list categories of typewriting errors. Perhaps the highest-level error is the capture error, i.e., the pattern gets captured by another but similar word ("efficiency" => "efficient"), which typically occurs at the abstract representation of the typing patterns in the graphemic buffer. As the graphemic buffer is providing output to both the typewriting and the handwriting motor system, one may expect these errors also in handwriting (Margolin, 1984). Another high-level error is the alternation reversal ("these" => "thses"), where the wrong, but maybe related movement is selected. One could hypothesize that these errors occur in the typewriting analogy of the allographic buffer.

The next lower-level typing errors concern the proper sequence of the keys. One could hypothesize that these errors occur in the typewriting analogy of the graphic motor-pattern buffer, i.e., the "typing motor-pattern buffer": transposition ("because" => "becuase"), and doubling ("Screen" => "Scrren"). Shaffer (1976) reports that transposition errors often occur when both hands alternate keys in an out-of-phase manner (e.g., "went down" => "wne todnw"), so that transposition errors are probably rarely found in handwriting. Rumelhart and Norman's (1982) model of the timing and sequencing of key strokes simulates exactly these types of errors. Their typing model will be explained at the end of this section.

2.6 Lower-Level Modules Reflected by Typewriting Movement Dynamics

After generating the typing motor-pattern code still various categories of "misstrokes" or "substitutions" may occur. The nature of these errors suggests that they could be produced in the module analogous to the parameter-setting module in handwriting. Grudin (1983) investigated video recordings of instances of these misstrokes and suggests that they are not just aiming errors but actually have been planned as a clear movement to the wrong key. The typing errors in experts and novices consist mostly of "horizontal misstrokes", i.e., hitting the wrong key in the same row as the correct one. Often the wrong finger was selected hitting its usual key. Less frequently occurring are "vertical misstrokes", where the appropriate finger moves in the wrong direction to the higher or the lower row of keys. Even less frequent, but still interesting are "homologous errors", where the corresponding finger of the opposite hand is activated. These errors indicate that wrist, hand, finger, and their movement directions do not need to be specified in a fixed sequence, but are independently substituted into a "matrix". Interesting is that the key stroke after a misstroke is delayed (Salthouse, 1984), which suggests that individual key strokes are monitored instead of producing whole groups of key strokes as "ballistic typing chunks".

The lowest-level typing error may be the intrusion or insertion error, resulting from a movement-aiming error, such that the finger lands at a position between two keys. Here, movement amplitudes in horizontal or vertical directions were not properly selected. This seems a typical error occurring in a module analogous to the parameter-setting module in the handwriting model. Of course, there are still many unclassified "misstrokes" and omissions in typewriting. However, the ones mentioned here seem to suggest at least close parallels between multi-module models of typewriting and handwriting.

2.7 Feedback in Handwriting

A serial-module model is appropriate to the extent that feedback loops between modules are not essential. This may indeed be the case in fast, single-phasic reaching movements. However, in continuous handwriting, not all parts of the writing pattern need to be prepared before movement can start, but rather subsequent parts of the movement may be prepared during producing the first part, while gradually correcting on the basis of earlier feedback. In his multi-module model, Ellis (1982) included feedback loops but he stated that they play a marginal role in fast, overlearned handwriting movements under stable execution conditions. The main reason is that handwriting is performed as fast as 100 ms per stroke, so that any central processing will be too slow to monitor stroke formation. For example, a sudden increase or decrease of pen-to-paper friction results in an immediate effect upon stroke size. It takes several strokes to restore stroke size (Denier van der Gon and Thuring, 1965; See Figure 5. Even the electromyogram (EMG) in fast movements seems at least during the first 100 ms independent of proprioceptive feedback as shown in conditions where the movement was blocked unexpectedly (e.g., Wadman, Denier van der Gon, Geuze, & Mol, 1979).

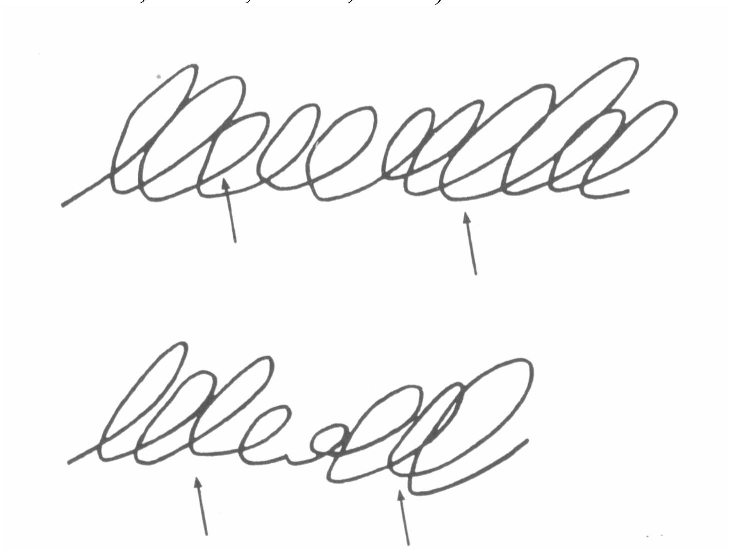

Figure 10.5: The influence of a sudden increase of the pen-to-paper friction (left arrows) and the restoration of the original pen-to-paper friction (right arrows). Several strokes of a t least 100 ms are required to recover from the distortion, which was not internally anticipated, [From Denier van der Gon and Thuring, 1965.]

Furthermore, the speed of writing seems little affected by the absence of visual feedback (Smyth & Silvers, 1987). Global parameters, such as instructed size changes are performed more accurately with than without vision, though (Burton, Pick, Holmes, & Teulings, 1990). Also distortion of visual feedback using rotation and shear transformations shows some global effects, although the subjects did not realize the type of distortion. Distorted orientation and slant show a gradual and partial correction towards the writer's usual orientation and slant (Pick and Teulings, 1983).

Although visual feedback is not used for monitoring the current strokes, it still appears that specific slips of the pen occur in the absence of visual feedback. When suppressing visual feedback, subjects appear to have a poor control of the number of stroke repetitions or letter repetitions (Lebrun & Rubio, 1972). Interestingly, patients with lesions in the right-hemisphere seem to show similar disruptions, e.g., "comptable" => "comnptabble," suggesting that the visual representation is used to monitor the writing process.

2.8 Serial-and-Parallel Model of Handwriting

Previously, the motor system was considered as a sequence of serial modules, where each module represents a specific process. In strictly serial modules a module needs to deliver its results, before the subsequent module can start processing. However, this seems unlikely in continuous motor tasks such as cursive script. Namely, during writing a sentence, the later parts of the sentence still may need to be prepared. Therefore, all modules are active simultaneously, i.e., in parallel.

Perhaps the earliest evidence for on-line programming was reported by Klapp and Wyatt (1976). They investigated a Morse task consisting of sequences of two key presses, in a "choice-reaction time" paradigm. It appeared that reaction time was lengthened in the more complex Morse patterns (i.e., short-long, or long-short) as compared to simpler patterns (i.e., short-short, or long-long). However, with practice the reaction time was less and less affected by the second key press. This indicates that much of the programming of the second key press is done during the first key press. Accordingly, both duration of the first key press and the interval between first and second key press were longer when the second key press is the more complex long one, as compared to the more simple shorter key press.

The phenomenon of slowed movement execution during loads due to preparation of later parts of the movement is also observed in "simple-reaction time" conditions, where the movement pattern has already been prepared to some extent and the subject is merely waiting for the "go signal" to start the execution. Sternberg, Monsell, Knoll, and Wright (1978) investigated speech and typewriting sequences in simple-reaction time conditions. By postulating a speech unit as a single stress group and a typewriting unit as a single keystroke, it appeared that both movement latency of the first unit and the execution time per unit increased about equally for each unit in the total sequence. The increase was about 10 ms per unit, although the effect reduces with exercise. Apparently the total number of units but not the complexity per unit is relevant for the movement latency. For example, the 10 ms increase in reaction time is independent of the number of syllables or connecting words per stress group (i.e., word complexity). Sternberg et al. suggested an exhaustive unit-retrieval process from an unsorted, non-reducing buffer followed by a separate unit-unpacking module. The buffer could be the analogy of the mid-level graphic motor-pattern retrieval in handwriting. The retrieval effects depend upon the number of major movement units and become manifest as delays before and during movement execution. The unit-unpacking module could be the analogy of the lower-level modules of parameter setting and motor initiation in handwriting. The unpacking effects depend upon the complexity of each unit separately so that movement latency is affected by the unpacking of the first unit only.

Interestingly, the increases of reaction-time and movement-time with the number of units are also observed if all (speech) units are identical: They seem even retrieved and executed at a significantly lower rate than if they were different units (Sternberg et al., 1978). The same was observed in handwriting for sequences of identical allographs (Teulings, Thomassen, & Van Galen 1983), and surprisingly also for sequences of identical syllables (Van Galen, 1990). Of course, in choice-reaction conditions, sequences of identical units are still initiated faster, probably because only one unit needs to be retrieved from the higher-level graphemic buffer (Teulings et al., 1983).

Initially the Sternbergian increase of both movement latency and movement duration as a function of the number of units in the string could not be replicated in handwriting, though (Hulstijn & Van Galen, 1983; Teulings, Mullins & Stelmach, 1986). It has been hypothesized that under time pressure, which is the case in these simple-reaction time experiments, subjects tend to preprogram largely the whole sequence rather than only the beginning. Van Galen, Meulenbroek and Hylkema (1986) eliminated time pressure by letting the subjects initiate and perform the writing pattern at their own pace after a go signal. Under these conditions, the Sternbergian movement-latency and duration effects as a function of sequence length in handwriting became manifest. A speculation why especially identical units cause an extra slowing down of movement execution is that these sequences give rise to specific errors. E.g., Lebrun and Rubio (1972) reported counting errors (under reduced visual feedback) in patterns containing repetitions. Furthermore, Rumelhart and Norman's (1982) typewriting model provides special provisions for adjacent and for nonadjacent occurrences of identical keystrokes (See end of this section).

A serial-and-parallel model has been proposed by Van Galen (1986, 1991) in order to explain the specific effects of parallel multi-module processing in handwriting. Serial stands for the notion that several sequential modules are discerned, ranging from the highly abstract intention of formulating a message to performing the appropriate muscle contractions. Parallel stands for the notion that all modules are active during movement production. In the multi-module model the output of the higher-level module is transmitted to the next lower-level module, so that the higher-level module is available to process subsequent parts of the writing movement. At the same time, the unit of programming at the high-level module may be large, whereas it is hierarchically divided into smaller low-level units (e.g., Povel & Collard, 1982; Rosenbaum, Kenny, & Derr, 1983; Sternberg, Knoll, & Turock, 1990). This suggests that in the higher-level module, more wide-range errors may occur than in the lower-level modules, and that it is reasonable to suppose that the reverse holds. Furthermore, the more abstract movement information is processed at the higher-level module, which needs to be done more anticipated with respect to its execution. In the opposite case, the more concrete movement information may be processed at the lowest level of the motor system, which can probably be done immediately prior to executing the corresponding movement.

The model to explain the differential effects due to the processing demands at higher and lower levels may be completed in two ways: According to one model, a higher-level module may take more processing time for a particular complication and consequently delivers results to the next lower module slightly later. This would imply that the corresponding part of the writing movement arrives at the lowest level with some delay. Thus, the execution of a complicating part of the writing pattern is delayed. Although reasonable, this is not what is observed. The observations rather support a second model, which says that the processing taking place in a module consumes central processing resources. This implies that execution of the current movement is slowed down at the very moment that higher-level processing is done, analogously to a timesharing computer system. The unique consequence of the latter model is that the higher the level of the module that produces processing load the more prior to outputting the corresponding part of the writing pattern. This becomes manifest as movement delays at a specific time prior to producing the complex part. Therefore, the opposite of the rationale seems reasonable: The more prior to the outputting of a complex part the delay is observed, the higher the level of the processing module involved.

Van Galen et al. (1986) tested the consistency of this hypothesis by proposing three complexity levels, i.e., the syllable, the allograph and the stroke structure, which are supposed to cause programming complexities at the levels of the graphemic, allographic and graphic motor-pattern buffers, respectively. Their subjects wrote pseudowords at their own pace after a go signal. Complexity at the syllable level was varied by adding letters after the third (target) letter (e.g., "feb" versus "febel"). This caused mainly a longer movement latency (286 versus 298 ms). The complexity at letter level was manipulated by increasing the complexity of the third letter itself ("l" versus "b", e.g., "fel", "felel", and "feleb", versus "feb", "febeb", and "febel"). This caused a longer duration of the letter immediately prior to the manipulated letter (270 versus 273 ms, which would be a small difference for reaction times, but is significant for movement times). The movement latency and the durations of the other letters were hardly affected. The complexity at the stroke level was manipulated by varying the connecting stroke between the second and the third allograph. Its complexity depends upon whether the connecting stroke has a constant sense of rotation (e.g., "ega") or a change of sense of rotation (e.g., "egu"). This caused a longer duration of the strokes of the allograph prior to the connecting stroke (397 versus 405 ms). In summary, these data support the notion that the higher-level modules consume processing demands in parallel to the lower-level modules, while the lower the level, the narrower-scale the influence and the smaller the unit of programming (See Table 1). Furthermore, repetition of a unit of programming causes slowing down of the execution.

Table 10.1: Programming unit, variable affecting complexity and the scope of the resulting delays, in various modules or buffers.

Module/buffer	Programming unit	Complexity variable	Delays
Graphemic	Word or stress group	Syllable structure	Word latency
Allographic	Syllable	Allograph structure	Previous allograph
Graphic motor-pattern	Allograph	Stroke complexity	Previous strokes

2.9 Serial-and-Parallel Model of Typewriting

The parallel preparation of successive key presses in typewriting has been suggested by many researchers as an explanation of the observed high typing rates of about 10 per second. There is ample evidence that this high speed of typewriting is only possible if several keystrokes are prepared in parallel. If only a preview of a single letter is given, typewriting slows down dramatically (Salthouse, 1984). Gentner, Grudin, and Conway (1980) and Larochelle (1982) showed that finger movements are not initiated according to the sequence in which they are going to press the keys, but start moving a variable time ahead, so that each finger arrives just in time at the right key. Indeed, Gentner (1983) states that, with practice, the sequential strategy of typewriting, is converted into a more and more parallel strategy, while the cognitive constraints are solved and more hard motor constraints remain. With practice, several fingers are observed to be moving simultaneously, each towards its respective target keys.

The parallel preparation of key presses has been captured in a model suggested by Rumelhart and Norman (1982). Although it is both a microscopic model and a computational model it is included in this section because it covers several of the modules discussed before, and because it illustrates the parallel processing introduced previously. The model generates inter-keystroke duration patterns which are so natural that they cannot be discriminated from the timing patterns generated by human typists. This is remarkable as the simulation was based on an isochronous timer with delays based the required on hand and finger movements. Furthermore, several types of human typewriting errors could be simulated. In this "Activation Triggered Schema" (ATS) model a schema is understood as a motor program, which in turn, may call subschemata, or child schemata, or subprograms, thereby passing specific parameters analogously to a computer-subroutine. These subschemata may also employ local processing of feedback from "extension detectors" of the finger relative to the target key. The master schema contains the letters to type a word and is analogous to the graphemic buffer in handwriting. This schema calls the subschemata to type the individual letters, which are analogous to the allographic buffer in handwriting. These subschemata, in turn, call lower-level subschemata controlling the finger, hand, and arm muscles until internal position feedback indicates that the appropriate finger has reached the planned key. The latter subschemata are analogous to the graphic motor-pattern buffer, parameter setting, and movement initiation in handwriting.

When starting to type a word, the subschemata of the constituent letters are activated according to their order in the words to follow. The model does not allow two identical letters to be activated simultaneously. If two identical letters are adjacent a "double schema" is generated with an activation just above the activation of the letter to be doubled. On the other hand, if two identical letters occur nonadjacently in the words to be typed, then the second occurrence and all following letters will be blocked until the first occurrence of the very letter has been executed and deactivated. Furthermore, in order to simulate motor noise some random variations are superimposed upon the activation levels (See Figure 10.6).

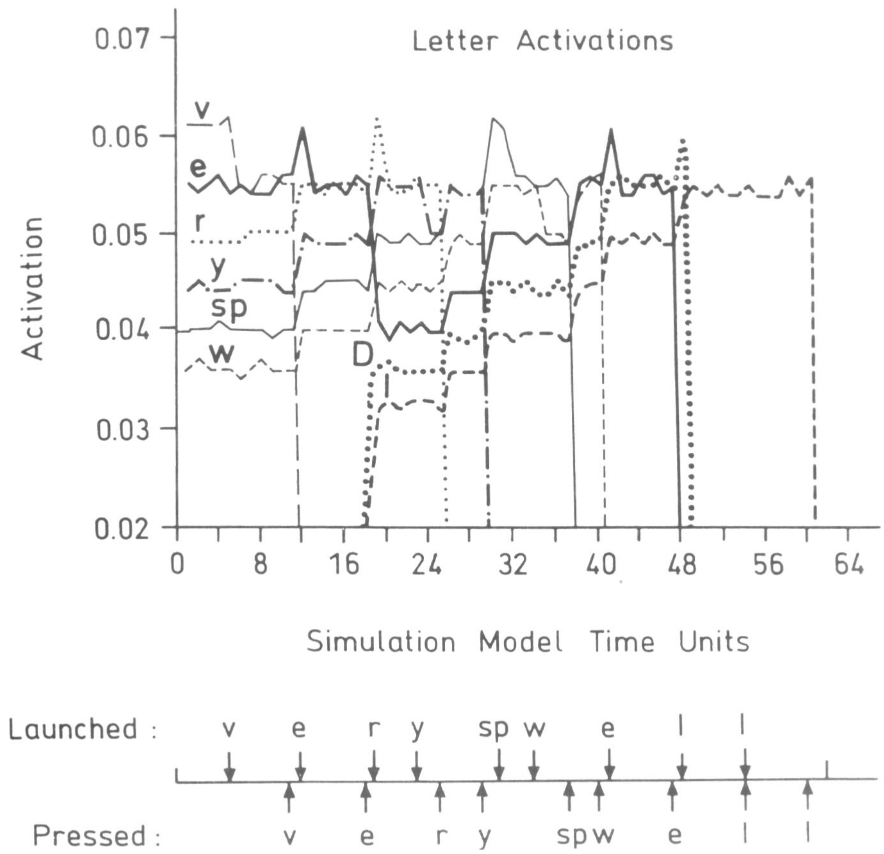

Figure 10.6: Patterns of activation of the key press schemata, required to set up the string 'very <space> w…'. As soon as the finger reaches the target key, the key press command is launched, After pressing the first 'e', the schema for the second key press of letter 'e', the double schema D, and the schema for the key press of letter 'l' are set up in order to complete the second word 'well'. [Adapted from Rumelhart and Norman, 1982.]

How do the fingers move towards the appropriate keys? Each letter schema obtains parameters describing the hand and finger that has to press the key. Furthermore, each finger has its own, specified movement limits towards upper and lower rows and inner and outer columns of keys. Therefore, also hand and arm movements need to be involved to reach some keys. The model hypothesizes that the hand is pushed and pulled towards the target positions of all the activated letter schemata in proportion to their activation. So the fingers and the hand are pulled towards the most activated schema, which represents the first key to be pressed. However, the other activated schemata pull the fingers and their hand to some extent also towards the subsequent letters, thus generating a smooth and anticipating movement of the hands and the fingers during typewriting.

The model suggests that some internal position feedback indicates that the finger, moving according to the activated schemata, has reached the key. A ballistic key press is launched, and after some time, a key, which is hopefully the correct one, is hit. The letter schema is then deactivated. If the letter schema with the highest activation has a double schema with still higher activation, then the letter is launched but it is the double schema which is deactivated so that the very letter is still activated and can be launched for the second time in succession. Deactivation is done till the activation level where the letter obtains its appropriate serial position, i.e., just below the activation level of its predecessor, or zero if the letter does not occur within the preparation window.

Due to the noise which is artificially superimposed upon the activation levels, several errors could be simulated. The activation of the double schema may increase beyond the next highest activated schema or decreased below the activation of the schema it has to operate upon so that the double operation may work upon the letter before or after the intended double letter, respectively, resulting in the usual double errors (e.g., "sheer" => "sherr"). Also transposition errors, even multiple ones between hands, could be simulated (e.g., "vitamins" => "ivtmaisn"). Furthermore, misstrokes could be simulated, occurring when during launching a key, hands and fingers move under the influence of the other, activated schemata. As a result, a key near to the intended one was hit (e.g., "awareness" => "awareneww"). The simulations produced no higher-level typing errors such as captures and alternation reversals or lower-level errors such as homologies because these would require additional provisions.

2.10 Empirical Evidence of Movement Units in Handwriting

The previous sections intended to establish several abstract processing modules. Each of these modules operates with larger or smaller units of programming of handwriting, typewriting, or speech. The elementary, mechanically based handwriting-movement unit, may still be the ballistic stroke, i.e., the trajectory between successive minima of the absolute velocity. However, there is little evidence that strokes form a higher-level organizational unit (Wing, 1978; Teulings et al., 1986; Hulstijn & Van Galen, 1988). The latter authors found that the unit of programming depends on the amount of practice. Unfamiliar and unpracticed patterns are considered as a sequence of stroke units, whereas highly practiced stroke sequences may form a single unit at the higher level. Van Galen (1991) suggests a gradual reduction of the units of programming when going from the higher-level modules to the lower-level modules: word, syllable, allograph, and stroke, respectively.

Teulings et al. (1983) compared whether a complete allograph or a single stroke forms a movement unit. Their subjects performed all pairs of the allographs "e", "u", "n", and "j" in a reaction-time paradigm with various kinds of advance information: both allographs, only the first allograph, only the second allograph, or no allograph. Usually, the allographs "e" and "u" are produced by counterclockwise strokes and "n" and "j" by clockwise strokes. It was hypothesized that, if strokes form movement units, then pairs like "eu" (called similar pairs) consist of a sequence of similar strokes and should show the same reaction times and movement times as identical pairs (e.g., "ee"). On the other hand, if allographs form movement units, then similar pairs should show the same data as nonsimilar pairs (e.g., "en"). Only the latter hypothesis appeared to hold. Pairs of identical allographs yield short choice-reaction times whereas both pairs of similar and nonsimilar allographs yield long choice-reaction times. Furthermore, identical pairs were executed at a lower rate in all choice-reaction and simple-reaction conditions. These data are compatible with the model that complete allographs form units.

There are more observations supporting the idea that each single allograph forms a unit. Meulenbroek and Van Galen (1989) found that the strokes belonging to an allograph are systematically different from the connecting strokes between allographs. The latter tend to be longer in time and size, and more noisy. Noise was expressed by the number, frequency and density of dysfluencies and by relatively more high-frequency components. Also between-allograph widths are more variable than the widths of the allographs themselves (Burton, Pick, Holmes, & Teulings, 1990). Even if connecting stroke and allograph stroke are virtually identical, differences have been found: Meulenbroek and Van Galen (1989) conducted a control experiment where adult subjects wrote several times a 6-letter word containing "uu". Here, the (within-allograph) upstroke of the "u" is virtually undistinguishable from the upstroke connecting both "u"s. Again, the connecting stroke was more variable. Another example is that with experience and age (from 8-year olds to adults) each writer acquires a handwriting style which is more and more individual and deviant from the original instruction method. Between each pair of allographs a different connecting stroke may exist, so that connecting strokes may be formed more freely than within-allograph strokes. Therefore, connecting strokes appear to become more easily individual (e.g., Sassoon, Nimmo-Smith, & Wing, 1989). Finally, because of their higher number of degrees of freedom, connecting strokes are more sensitive to withdrawal of visual feedback: Between-subjects differences of connecting strokes increase more than those of strokes within an allograph (Meulenbroek & Van Galen, 1989).

In this context it is interesting to mention results by Maarse and Thomassen (1983), showing that the directions of the up strokes, which form mostly connecting strokes, are less stable both within and between conditions. The conditions were formed by instructing the subjects to write normal, at increased width, or at decreased width. In summary, most empirical evidence supports the allographs as a higher-level unit of handwriting, but apparently this does not reject the hypotheses of bigger or smaller units at higher or lower levels or under certain conditions of practice, respectively.

2.11 Empirical Evidence of Movement Units in Typewriting

It seems reasonable to consider a keystroke as a unit in typewriting, because it is so discrete as compared to the fluent and continuous handwriting movement. Indeed Sternberg et al. (1978) found similar affects in keystrokes as in stress groups, which seemed to fit the properties of a movement unit best. Typewriting strokes are produced at about the same rate as handwriting strokes. However, several handwriting strokes form higher-level allograph units, so that one may wonder whether there are higher-level movement units of several typewriting strokes. Logan (1982) found that when subjects received a stop signal during their typewriting, they could inhibit the very last letter of a word very well. This indicates that the unit in typewriting is smaller than a word, and that a key stroke is still a useful unit in typewriting.

3 MICROSCOPIC MODELS

In the previous section the complex handwriting motor system has been segmented into various serial-and-parallel modules. An attempt was made to integrate also the typewriting motor system. In the present section specific modules and processes will be discussed in a more detailed, microscopic scale. The highest-level module concerns the graphemic module, which outputs both to the lower-level modules of the handwriting, typewriting, and speech motor systems (Ellis, 1982; Margolin, 1984). The graphemic code is translated into the allographic code, which is unique for each of these modalities. The allographic code prescribes the shapes of the graphemes in terms of the strokes required but not the sequence or execution directions of the strokes. This information is retrieved from the graphic motor-pattern store when translating the allographic code into the graphic motor-pattern code. The sequence and directions of the strokes are mostly chosen according to some rules, summarized in the "grammar of action". The translation between these buffers will be discussed first. The module to be discussed next concerns graphic motor-pattern store. Not only movement sequences and directions are stored here, but also other motor information to the extent that it is not substituted by the lower-level modules of the non-muscle-specific parameter setting or the muscle-specific motor initiation. The fixed motor information could be understood as a motor program. The third module to be discussed concerns the lowest-level motor initiation module, where the motor system is dealing with muscle- (or limb-) dependent parameters. Strokes in different directions require different sets of muscle-limb systems, having different features, resulting in main axes of movement in handwriting. Finally, an overview is presented of all conditions and modules contributing to the time required to produce specific movement times. The large number of factors affecting movement time seems an indication that durations are not stored in the graphic motor-pattern but are generated by various modules.

3.1 Grammar of Action

As explained in the previous section on macroscopic models, the allographic code is retrieved from the allographic store and temporarily stored in the allographic buffer. Confusions between allographs having similar shapes but different stroke sequences, made Ellis (1982) hypothesize a module where the movement code specifies only the shape of the pattern but not its stroke sequence. The sequence of strokes is established after retrieving the graphic motor-pattern code. If familiar writing patterns are produced the sequence of strokes is fixed and well-learned. However, if new patterns have to be produced, an efficient sequence will be chosen on the spot, which appears to satisfy a limited set of rules. This set of rules is known as a "grammar of action", originally proposed by Goodnow and Levine (1973), and extended for handwriting by Van Sommers (1984), Thomassen, Tibosch, and Maarse, (1989) and others.

A graphical pattern may in principle be produced according to numerous sequences and directions of the constituting strokes. Thomassen et al., (1989) indicated that the number of different stroke sequences to copy a geometrical pattern increases explosively with the number of segments n as $n!\ 2^n$. However, reading habits, writing instructions, biomechanical effector properties, opportunity for visual guidance and efficiency lead to a reduction in the number of sequences to a few usual sequences, which are chosen by most subjects. The grammar of action specifying preferences to perform the segments may also be the compound result of maturation of the motor system and handwriting skill as several developmental trends and transitions have been observed (Goodnow & Levine, 1973; Nihei, 1983; Thomassen & Teulings, 1979, 1983b; Thomassen, Tibosch & Maarse, 1991; Van Sommers, 1984).

The order of stroke production in simple geometrical patterns composed of straight horizontal and vertical segments was investigated systematically by, e.g., Thomassen, Meulenbroek, and Tibosch (1991). They observed that five weighted rules could predict nearly 88% of the observed production sequences correctly. These rules are in decreasing order of their weights:

(1) Threading (i.e., continuing without pen lift).
(2) Starting at the left extreme.
(3) Anchoring (i.e., lifting the pen and repositioning it at a segment drawn earlier).
(4) Starting with verticals.
(5) Starting at the top.

Some patterns require trading-off conflicting rules. E.g., Rule 3 is traded off with Rule 1; Rule 5 is traded off with Rule 2. The stroke sequence of patterns where all rules are satisfied simultaneously, were less variable, and both reaction and movement times were shorter than in patterns requiring choices between conflicting rules. In patterns requiring tradeoff between various rules, the latencies are longer while the movement times are not longer but even shorter. Therefore, ample time before actually producing the pattern, these very high-level decision processes consume central resources (Thomassen, 1991; Thomassen et al., 1991). After extended practice the sequence of strokes of a pattern becomes established as a time-ordered structure in the graphic motor-pattern store.

3.2 Motor Program

Various definitions of a motor program may exist. It is generally accepted that a motor program consists of an abstract memory structure containing codes capable of being transformed into movement patterns (Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979). A motor program is a controlled sequence, produced by hierarchical units (Keele, Cohen, & Ivry, 1990), which is somehow stored in a person's long-term motor memory. As the sequence of strokes is represented in the motor program, the motor program seems most compatible with the graphic motor-pattern code.

Handwriting, typewriting, speech, gesture, and gait patterns are indeed highly personal (e.g., Cutting and Kozlowski, 1977; Schmidt et al., 1979). Especially handwriting and signatures are considered as personal, motoric finger prints (e.g., Stockholm, 1979; Rosenbaum, 1991). The idiosyncrasy of handwriting is not due to each person's specific handwriting instruction as children receiving the same writing instruction soon develop their personal handwriting. The sources of personal handwriting features are rather the individual storage of handwriting movements in the graphic motor-pattern store ("invariant-storage hypothesis"), or, the individual training of the lower-level movement processing modules ("invariant-process hypothesis"). The individual training could result in individual movement features caused by the muscle-independent parameter setting or the muscle-dependent motor initiation (e.g., Van Emmerik and Newell, 1989; Soechting, Lacquaniti, & Terzuolo, 1986). Some global handwriting features, generated by those lower-level modules, appear so individual that they can be used to identify a writer by analysing an arbitrary sentence of handwriting. A discriminative set is formed by some global, dynamic features: mean axial pen pressure (i.e., axial pen-paper pressure), mean absolute pen velocity in downstrokes, sum of the pen-down durations divided by the total writing time, and by some global, static features: handwriting slant, and horizontal extent of the strokes (Maarse, Schomaker, & Teulings, 1988).

Nevertheless, there is strong evidence for the invariant-storage hypothesis. For example, handwriting storage seems hierarchically organized (e.g., Patterson & Wing, 1989). Hierarchical storage may also explain why related allographs in adult handwriting show similarities, e.g., between the descenders in "g" and "y" or the ascenders in "l" and "b". These complex stroke similarities seem unlikely to be caused by the lower-level processes. The invariant-storage hypothesis can also explain the "motor equivalence" phenomenon (Bernstein, 1967), that the shape of handwriting patterns is rather independent of the limbs and muscles involved. There is a striking shape resemblance between a person's handwriting pattern of normal size and the same pattern performed much larger, or when using other limbs, such that different muscles are involved (Bernstein, 1967; Denier van der Gon & Thuring, 1965; Katz, 1951; Keele et al., 1990; Lacquaniti, 1989; Marsden, 1982; Merton, 1972; Raibert, 1977; Stelmach & Teulings, 1983; Teulings et al., 1983; Wing, 1990; Wright, 1990; etc.). In terms of the multi-module model this can be understood by pointing out that writing patterns are stored independently of muscles and limbs, whereas the muscle-specific aspects are substituted in the lowest-level motor-initiation module.

If a motor program is repeated several times, the invariant-storage hypothesis predicts many invariant movement features across the resulting replications. By reversing this rationale, which is reasonable, invariant features might stem from those movement parameters that are stored in the motor program, i.e., the graphic motor-pattern code. Additional consistency of invariant features may help to provide further evidence that certain movement parameters are stored in the motor program. Additional consistency can be provided by showing that a supposedly invariant feature does not change when irrelevant execution conditions are changed.

The identification of invariant features is the key to estimating the underlying motor parameters of the motor program. Therefore, invariances have to be expressed on a common scale so that they can be compared. The invariance of a certain feature can be expressed by its signal-to-noise ratio (SNR), defined here as the ratio of the standard deviation of the mean stroke pattern sd(signal') to the mean standard deviation of the differences between the individual stroke patterns and the mean stroke pattern sd(Noise). Actually, an unbiased estimator for the noise standard deviation is

$$\mathrm{sd(signal)}^2 = \mathrm{sd(signal')}^2 - \mathrm{sd(noise)}^2/n$$

where n equals the number of replications used to estimate the mean, although this may yield negative estimates (e.g., Teulings, & Schomaker, 1993). Any overall variations cause overestimation of sd(noise) so that these small overall variations need to be normalized by a factor, although a rate parameter for time, for example, does not seem statistically reliable in typewriting (Gentner, 1987). However, these normalizations affect the data only little, so that the effect of a non-reliable rate parameter is minor. The SNR of a feature presents a dimensionless measure of the invariance of a feature. Furthermore, 1/SNR\s2\S represents the relative noise variance of a feature, and is additive for products of independent features, just like absolute variances of sums or differences of independent features are additive. Therefore, not only absolute measures of invariance can be given, but also, how invariances of various features relate to each other.

If it is known how certain features are related, more conclusions can be drawn. A feature could be invariant solely because this feature is related to another and possibly more invariant feature. The comparison of various related invariant features can be illustrated by considering the simplest, one-dimensional mechanical equation, which expresses stroke size s, as a function of the net force level, which may be assumed proportional with the acceleration a in the friction-less case, and stroke duration t:

$$s = eat^2$$

where e stands for the efficiency of the force-time curves. The efficiency is proportional with the distance travelled given a certain peak force and duration and is highest for a block-shaped acceleration-time function, and less for half cosine or triangular-shaped acceleration time-functions, namely 0.25, $2/\pi^2 \approx 0.20$, and $1/6 \approx 0.17$, respectively. The small dynamical range of the efficiency e as a function of acceleration shape indicates that it is not a powerful parameter to control movements.

It is often observed that timing patterns of strokes are remarkably invariant across replications (e.g., Viviani & Terzuolo, 1980; Denier van der Gon & Thuring, 1965). However, stroke duration and peak force per stroke are negatively correlated so that it can be understood that the resulting stroke size s is more invariant than either of its factors a and t (e.g., Teulings, Thomassen, & Van Galen, 1986). If the a and t features would be generated independently by the motor program, their relative noise variances would add to the relative noise variance of the s feature. However, the s feature shows smaller relative noise variances than the a or t features. This is compatible with a negative correlation between the a and t features. Furthermore, if global writing time or if writing size are deliberately varied by the subject, the s feature appears to change in a more predictable way according to a linear transformation than the a and t features. These two findings suggest that the s feature is more closely related to an underlying movement parameter of the motor program whereas the a and t features are rather related to movement parameters derived at lower levels. In other words, the spatial structure in terms of the relative stroke sizes are more likely to be represented in the graphic motor-pattern code than stroke durations or force levels. This is indeed what one would expect in motor tasks such as handwriting, where the task is specified in the spatial domain and not in the temporal domain.

Someone's signature is generally accepted as a motoric finger print. The problem of signature verification or identification is interesting to the extent that it requires searching of those parameters that are probably stored in the graphic motor-pattern code and that may be both difficult to suppress or to imitate (e.g., Lorette & Plamondon, 1990). For that reason, not only the (static) spatial structure, which may be stored in the graphic motor-pattern code, but also the pen pressure as a function of time seems an interesting dynamical feature for automatic signature verification (e.g., Crane & Ostrem, 1983). Apart from the uniform, systematic increase of pen pressure towards the end of a word (e.g., Kao, 1983), Schomaker and Plamondon (1991) state that axial pen pressure seems a centrally stored pattern, as the pen-force pattern is relatively invariant and not related to the allograph shapes or any biomechanical effects. Because of its unclear relation with allograph shapes, pen pressure is of little use for automatic, on-line handwriting recognition, though.

Newell and Van Emmerik (1989) recorded the vertical component of the pen-tip position and simultaneously the vertical movement components of the hand, wrist, elbow and shoulder joints. They studied right-handers, who wrote with their dominant and with their nondominant hands, and analogously left-handers. The two subject groups employed different coordination patterns in terms of the correlations between joint angles, when writing with their dominant hands. These coordination patterns are again different for loops, circles (i.e., loops without left-to-right movement), or signatures (Emmerik & Newell, 1989; Soechting et al., 1986; Lacquaniti, Ferrigo, Pedotti, Soechting, & Terzuolo, 1987). An interesting phenomenon is that both groups of subjects could transfer their invariant coordination patterns to the nondominant side. This means that left- and right-handers using their nondominant limb still do not resemble right- or left-handers, respectively. Important is that the multi-joint coordination patterns seem acquired strategies, which have been stored in some lower-level central module, and that they are not due to the neurobiomechanical hardware. This seems to support the invariant-processing hypothesis.

Van Emmerik and Newell (1989; 1990) realized that the left-to-right writing habit may be responsible for the very different coordination patterns observed in left and right-handers. Indeed, when performing tasks without left-to-right translation (e.g., circles) the coordination patterns between left and right-handers become more similar. Writing habits are apparently influencing the way handwriting movements are produced. This was also found by Thomassen and Teulings (1979), who observed counterclockwise preferences for both left and right hands in school children and adults when performing drawing patterns (e.g., circles, triangles). These preferences may come from the handwriting habit to form cursive-script letters counterclockwise. Accordingly, children of the age of 5, who were not yet exposed to handwriting, showed anatomically-based preferences. (e.g., mirror symmetry for both hands). However, in fast, uncontrolled scribbling movements, as well as continuous circles of maximum speed, anatomical preferences of the writing apparatus play a predominant role in all age categories. Although handwriting patterns are the result of abstract motor programs stored in the graphic motor-pattern buffer in a muscle and limb-independent way (e.g., Klapp, 1977), whereas the latter parameters are substituted at the lower-level modules, this does not mean that there are no anatomical factors involved. Muscle and limb-specific effects play a minor role or are corrected for in the motor program. In fact, the latter examples show that anatomical factors and habits may become manifest under certain conditions.

3.3 Main Axes in Handwriting

Muscle- and limb-specific parameters are substituted in the lowest-level module: the motor-initiation module. This module has to deal with the asymmetric properties of the handwriting apparatus. As the handwriting apparatus has no rotational symmetry, its properties are likely to depend upon movement direction. Indeed, back-and-forth movements generated by flexing and extending the finger joints take about 1.3 times more time than those by the wrist-joint (McAllister, 1900; Teulings, Thomassen, & Maarse, 1988). Wrist-joint movements and the finger-joint movements correspond to nearly orthogonal directions or main axes each with characteristic properties: wrist-joint movements are fast and finger-joint movements are slow, whereas movements in intermediate directions have intermediate stroke durations. The idea of main axes is not new. Denier van der Gon and Thuring (1965), referred to Dal Bianco (1944), who found that movements along the more horizontal axis were disturbed in cerebellar-injury patients, whereas movements along the perpendicular axis were not. The latter author concluded that movements can very well be organized in body-oriented orthogonal axes. Some authors (e.g., Dooijes, 1983; Plamondon and Lamarche, 1986; Maarse et al., 1986) define "subjective main axes" by asking the subjects to produce voluntary wrist-joint movements and finger movements without wrist movements. These main axes are often oblique. Dooijes (1983) called them "principal directions", and supposed that they moved at constant speed from left to right during writing. He found that the subjects were able to reproduce their voluntary main axes accurately, even after one year. Plamondon and Lamarche (1986) and Maarse, Schomaker and Thomassen (1986) estimated main axes ("principal axes" or "natural axes") in a similar way. Its directions varied relatively little as a function of arm rotation, e.g., when writing from left to right across a line, which suggests that these preferred directions are not purely biomechanically determined.

In order to understand the direction-dependent characteristics of the handwriting apparatus, its biomechanical structure is analysed in terms of degrees of freedom. Each joint of the handwriting apparatus is either a hinge-like or a universal joint. A hinge-like joint requires specification of only one angle (one degree of freedom), whereas a universal joint requires specification of two angles (two degrees of freedom). Neglecting the two universal joints of elbow and shoulder, which are actively involved in handwriting (e.g., Van Emmerik & Newell, 1989; 1990), the hand-finger system has in total at least ten degrees of freedom. Namely, the wrist joint possesses two degrees of freedom (dorsal/palmar flexion and ulnar/radial abduction). The thumb and the index finger each possess four degrees of freedom: one for each of the two peripheral finger joints (flexion/extension), and two for the proximal one (flexion/extension and adduction/abduction). The other fingers do not move independently from the index finger and therefore do not contribute to the degrees of freedom of the handwriting apparatus. However, not all passive or theoretical degrees of freedom are used in handwriting. In order to assure the necessary pen grip, the thumb and fingers are kept opposed. Furthermore, the pen is not supposed to rotate around its axial axis. Finally, the pen tip remains in touch with the paper during cursive writing. Thus the wrist joint has only one effective degree of freedom, formed by a fixed combination of palmar flexion and radial abduction/dorsal flexion and ulnar abduction, depending upon supination/pronation of the forearm. The thumb-and-fingers system has two, or maybe more, effective degrees of freedom. One component represents the back-and-forth movements to and from the hand palm by flexion/extension of both thumb-joints and finger-joints and, the other component represents the back-and-forth movements parallel to the hand by simultaneous flexion/extension of thumb-joints and the extension/flexion of finger-joints. Therefore strokes in intermediate directions use three degrees of freedom.

This analysis in terms of degrees of freedom gives rise to an interesting prediction about the accuracy of movements in various directions. Wrist movements have one degree of freedom and allow the near perfect production of large circle segments (e.g., with radius 15 cm), which can well be approximated by straight segments in normal stroke lengths of 0.5 cm (Dooijes, 1983). Therefore, trajectory inaccuracy can be expressed by the average distance of each stroke from its minimum-squares fitted line. However, finger movements have two degrees of freedom and therefore, require coordination between the two (or more) synergistic muscle systems. For example, it is only possible to draw a straight line if the two synergistic muscle systems are initiated simultaneously, in spite of unequal nerve transmission delays. Uneven initiation causes loops or blunt curves at stroke endings (Schomaker et al., 1989). Furthermore, the force-versus-time patterns need to be proportional. If not, a curved stroke results. Therefore, movements with two or more effective degrees of freedom produce less accurate pen trajectories. Finally, intermediate movement directions have three degrees of freedom and produce even less accurate trajectories than the pure finger movements. So the wrist and the finger movement directions can be identified by their capability to produce relatively straight back-and-forth movements. These special movement directions, which have relatively high spatial accuracies, appear to coincide with the directions of extreme stroke durations and also with those of extreme stroke lengths (Teulings, Thomassen, & Maarse, 1989). Table 2 summarizes the directional properties of the writing apparatus. Opposite to the accuracy of the straightness is the accuracy of the movement amplitude: Wrist movements are less accurate than finger movements, as established in Fitts' speed-accuracy tradeoff tasks (Langolf et al., 1976) (See Computational Models for a discussion on Fitts' law). The net difference seems small in typical handwriting movements having durations of about 140 ms, but increases for precision movements requiring more than about 200 ms.

Table 10.2: Various features and properties, which may differ for wrist and finger movements, and which may either have an intermediate or cumulative effect for movements of the wrist and fingers combined

	Joints involved		
Feature	**Wrist**	**Wrist and finger**	**Finger**
Direction(degrees)*	+45	Intermediate	-45
Preferred duration (ms)	120	Intermediate	160
Preferred size (mm)	9	Intermediate	6
Degrees of freedom	1	3	2
Accuracy of straightness (mm)	0.04	>0.07	0.07
Fitts' speed-accuracy (ms per bit)	43	?	26

*Relative to baseline (0 degrees); slant is +70 degrees' most frequent upward movement is +45 degrees.

An interesting is the finding that both movement axes are orthogonal and have average directions of +45 and -45 degrees with respect to the baseline of handwriting. The average slant of handwriting was 70 degrees relative to the baseline, and did not coincide nor correlate with any of the main axes. This is in agreement with the observation that slant as well as the directions of the subjective main axes appeared highly independent of arm orientation which changes when writing across the line (Maarse et al., 1986). Slant was estimated by the most frequent (opposite) direction of down strokes. Although visual slant was actually a few degrees steeper, this estimator was robust with respect to instructed stretching of writing (Maarse and Thomassen, 1983). Only the most frequent upward stroke direction, representing the connecting strokes between allographs, coincides more or less with the direction of the wrist-joint extensions (Teulings, Thomassen, & Maarse, 1989).

3.4 Factors Influencing Movement Duration

The preferred frequency of the handwriting apparatus appears to be close to 5 Hz (Teulings & Maarse, 1984; Maarse et al., 1986; Teulings et al., 1989). The speed of handwriting in terms of strokes per second varies with the maximum speeds of several, different motor tasks involving other muscles, such as tapping with the finger, thumb, hand, arm, or foot (See Keele & Hawkins, 1982, for an overview). The latter authors suggest that a central time keeper with unpredictable transmission delays could form the limiting factor for these speeds. On the other hand, Morasso et al., (1983) and Van Galen & Schomaker (1992) suggest rather that the inertia and the peripheral nerve-muscle system may act as a biomechanical low-pass filter of the higher-frequency nerve signals. At least there seems a physiologically determined maximum speed of repetitive movements. Previously, various conditions have been discussed, causing processing loads at the higher-level modules of the handwriting motor system, and which become manifest as small delays:

(1) On-line preparation of complex syllable structures and, syllable repetition.
(2) Allograph selection, allograph complexity, and allograph repetition
(3) Stroke sequence selection within an allograph.
(4) On-line preparation of complex connecting strokes.
(5) Direction of a stroke.

These factors cause only marginal movement-duration effects. The major effects upon movement duration are due to relative sizes and shapes of the strokes. The traditional view (Denier van der Gon & Thuring, 1965) is that the average duration of handwriting strokes does not depend upon their average size ("isochrony"). The principle of isochrony can be extended to the sub-stroke level ("isogony"). Isogony says that trajectories of equal change of direction are performed in equal amounts of time (Viviani & Terzuolo, 1980; 1982). This, in turn, implies at the microscopic scale, that angular velocity ω of the handwriting trajectory is constant, or, that pen speed v is directly proportional with the radius of curvature r: $v = \omega/r$. However, more refined predictions are possible. In general, the above approximations do not hold in extreme writing sizes, e.g., on a blackboard (e.g., Thomassen & Teulings, 1983b), in more complex patterns consisting of strokes of different sizes (e.g., Thomassen et al., 1986) or in curvilinear strokes (e.g., Lacquaniti, et al., 1983). In order to bring these different and conflicting relations into consistence, three context levels have been discerned by Thomassen and Teulings (1985). In each context, specific time versus size relations appear to exist. The compound effect is the product of three factors, because these three context levels form concentric shells:

(1) Macro context (i.e., a word in the context of other words of different overall sizes).
(2) Meso context (i.e., a single stroke in the context of other strokes of different sizes).
(3) Micro context (i.e., the local curve radius in the context of the curvilinear trace of a single stroke).

In macro context complete writing patterns are considered in isolation. As mentioned earlier, in the frictionless case, stroke size s is proportional to the peak muscle force (or peak acceleration a) and the stroke duration t squared: $s = eat^2$, where e is an efficiency factor, characterizing the effect of a particular shape of the force-time curve. The efficiency e plays a minor role in controlling stroke sizes in handwriting (e.g., Teulings et al., 1986). The shapes of the force-time curves are indeed rather constant (Plamondon & Maarse, 1989).

It appears that the time to produce a writing pattern is virtually independent of its size, provided that the average stroke sizes are not too big, i.e., between 0.25 and 1 cm (Michel, 1971; Denier van der Gon & Thuring, 1965; Thomassen & Teulings, 1984; though not in agreement with Wing, 1980). The duration is apparently limited by the frequency bandwidth of the handwriting apparatus so that size increase is entirely produced by force increase. Consequently, duration t is constant in the range of normal writing sizes so that peak force a is proportional with stroke size s. On the other hand, in large writing sizes, i.e., much larger than 1 cm, like when writing on the black board, force levels may become very high and may reach a ceiling. Thomassen and Teulings (1984) found that the height of this ceiling depends on the instructed pace or time pressure. Therefore, when producing large writing sizes or arm movements, it is the force level which seems constant, whereas size variations are programmed entirely by variation of duration (e.g., Wadman et al., 1979). Thus in macro context, a power relation between t and s can be proposed:

$$T = ks^b \qquad \begin{array}{l} (b = 0 \text{ if } s < 1 \text{ cm}) \\ (b = \tfrac{1}{2} \text{ if } s >> 1 \text{ cm}) \end{array}$$

where k is constant per context and instruction.

Apart from the force level approaching a ceiling there may still be another mechanism causing larger writing patterns to be produced slower, suggested by Keele (1982). Namely, relative accuracy is not necessarily kept under control when instructed to write larger. The most natural assumption would be that the subjects keep a constant relative accuracy so that writing patterns of various sizes appear very similar. For a large range of movement amplitudes, Fitts' law reliably predicts that aiming-movement time depends only upon the relative accuracies (See Computational Models for a discussion on Fitts' law). This would suggest that movement time does not depend upon movement size. However, when writing size increases, the arm gets more involved and the fingers less (Meulenbroek et al., in prep.). The critical observation is that arm movements require more time for a specific relative accuracy than hand or finger movements, namely 106 ms/bit, versus 43 or 26 ms/bit, respectively (Langolf, 1976). This holds for stroke durations longer than about 200 ms, which may occur in precision movements. In order to maintain the required relative accuracy also in larger writing sizes, where mainly arm movements are involved, the motor system may deliberately choose a lower rate. This hypothesis seems to be supported to some extent by data presented by Wright (1991). He showed that writing with the arm was indeed slower than writing with the fingers within a small ascender-to-descender size range of about 1.5 to 2.5 cm, although the reliable difference was as small as 5 ms/stroke. However, there are some small inconsistencies in this hypothesis. For example, writing time as a function of size of the two effector systems did not seem to interact (Wright, 1991), whereas the speed-accuracy relations clearly do (Langolf et al., 1976). Furthermore, finger movements, showing the highest accuracy, are not the fastest movements (See Table 2). Another difficulty to disentangle finger and wrist movements versus arm movements is the "hysteresis" observed, i.e., when increasing or decreasing writing sizes, the effector systems already involved tend to continue their contribution. Therefore, the relative contribution of these effector systems may depend upon the movements immediately done before.

Meso context refers to the adjacency of strokes of different sizes (such as in the cursive-script word "pellet"). The specific problem for the motor system is the abrupt adjustment of peak force or stroke duration, leading to a size increase from the "e" to the "l". Denier van der Gon and Thuring (1965) suggested a pure time increase with no increase of peak-force level, like in large movement sizes in macro context, but most observations are compatible with a increase of both time and force. This seems likely because a pure increase of peak force, without slowing the movement, may take several strokes (e.g., Denier van der Gon & Thuring, 1965; Stelmach & Teulings, 1983). Thomassen and Teulings (1985) suggested the following power function to describe the relation between duration t and size s in meso context:

$$t = ks^b \quad (b = +- 0.33)$$

The data of "el" or "le" pairs by Thomassen and Schomaker (1986), Thomassen and Teulings (1985), Hollerbach (1981), Greer and Green (1983), Wing (1980) yield values for b = 0.22, 0.34, 0.34, 0.38, and 0.41, respectively. Indeed, there is only marginal consistency, which may mean that b depends upon the subject and upon various unknown conditions. At least, b seems clearly within the range of the extreme values of 0.0 and 0.5, occurring in macro context.

Finally, micro-context describes the time needed per infinitesimal part of the writing trajectory, which is equivalent with the local pen speed v. An estimator of "local size" is curve radius r of the circle, fitting the curve at that point. An estimator of "local duration" is the inverse angular velocity, i.e., $r/v = 1/\omega$. Lacquaniti et al. (1983) observed that in a large variety of drawing tasks, a more complex relation holds than the one suggested by the isogony principle, namely:

$$1/\omega = r/v = kr^b \quad (b = 2/3) \quad (\text{"two-third power law"}),$$

where k is a constant gain factor which depends upon meso and macro context. This relation can be derived for perfectly sinusoidal movements, i.e., two equal-frequency movement components without left-to-right translation, which produces only arbitrary ellipses. The relation holds to some extent for a narrow frequency-bandwidth movement, consisting of two independent movement components (e.g., horizontal and vertical) which are piece wise sinusoids, such as in large, fluent scribbling movements. However, the two-third power law does not hold for normal handwriting. Thomassen and Teulings (1985) studied the behavior of the value b and the correlation between log(r/v) and log(r) for various writing patterns and simulated patterns. A normal handwriting pattern yields virtually the same values as a random-walk pattern: b = 0.41 (instead of 2/3) and correlation 0.83 (instead of 1.0, ideally). Also alternating small and big loops (e.g., "elel") yield values close to those of a random walk. Only continuous ellipses and "llll" patterns yield values which are closer to the ideal value than to those of a random walk: b = 0.59 and correlations as high as 0.95. The reasons that the two-third power law does not seem to hold for handwriting in general is probably due to the wide frequency band of handwriting (Teulings & Maarse, 1984), and the left-to-right trend, although this trend is probably only taking place during the upstrokes (e.g., Thomassen & Teulings, 1983a; Maarse & Thomassen, 1983). Wann et al. (1988) also observed significant deviations from the two-third power law in big, repetitive ellipses, produced at lower than maximal rates. The velocity patterns become skewed, due to faster accelerations than decelerations. Therefore, they suggested a "wobbling-mass model", which will be discussed in the next section on computational models. In summary, the two-third power law does not seem to describe the instantaneous speed of handwriting strokes accurately enough.

The observation that so many parameters affect movement durations provides additional evidence that durations are not stored in the graphic motor-pattern code (e.g., Teulings et al., 1986). Instead, the pattern of durations is the result of various time-consuming actions (e.g., Lacquaniti et al., 1983), which mainly take place at the lower-level modules of parameter-setting and motor-initiation. A similar line of evidence exists for typewriting. There appears no global typing-rate parameter (Gentner, 1987). The inter-key intervals are rather the result of fixed actions required to move to each subsequent key, as Rumelhart and Norman (1982) have demonstrated in their typewriting simulation model. The more complex handwriting motor system has not yet been captured by an extended model. There is a class of computational models of the writing movement which employ only a limited set of features of the writing pattern itself in order to reconstruct an existing writing pattern. However, the symbolic computational models may allow more than only reproducing existing writing patterns by substituting parts of the writing patterns by symbolic units, so that also new writing patterns can be generated. None of the computational models is so detailed that also these central and peripheral effects can be generated, though.

4 COMPUTATIONAL MODELS

Computational models focus on the quantitative simulation of the pen movements in handwriting. The aim is to generate handwriting movements with limited sets of parameters, which may relate to certain neurophysiological variables of the motor program or the biomechanical architecture of the handwriting apparatus. These models may at least suggest the validity of certain models, although it is impossible to draw firm conclusions. Furthermore, the models may indicate the complexity of the handwriting motor system.

4.1 Theoretical Minimum Number of Parameters

According to the sampling theorem (e.g., Jerri, 1977), the minimum number of (isochronous) samples required to reconstruct a time function of duration T and limited frequency bandwidth W, equals 2WT. This requires sampling of the handwriting movement at the Nyquist frequency, which is equal to 2W. Handwriting consists actually of two time functions, namely one for the horizontal and one for the vertical component. Under the worst-case assumption that both components are independent, 4WT parameters are required to exactly describe a handwriting segment of duration T. The frequency spectrum of handwriting shows a predominant frequency of 5 Hz and a gradual descent to noise level at about W = 10 Hz (Teulings & Maarse, 1984). The durations of the fastest ballistic strokes, which are easiest to simulate, are about 0.1 s. Therefore, 4 parameters per ballistic stroke allow the exact description of the handwriting movement in space and time. A more realistic estimate of the "equivalent bandwidth" of the non-homogeneous frequency spectrum is slightly smaller than the maximum bandwidth, namely typically W = 7 Hz. This explains why lowpass filtering of frequencies lower than 7 Hz deteriorates the shape of a handwriting pattern (e.g., Teulings & Thomassen, 1979). The average duration of ballistic strokes is slightly longer than the minimum duration, namely typically T = 0.14 s. Again, on average, 4 parameters per stroke are required to reconstruct the writing signal completely.

Perhaps the most unintelligent computational model describing the handwriting movement adequately would sample isochronously and simultaneously (and infinitely accurately) x and y coordinates at the Nyquist frequency f = 2W = 2/0.14s = 14 Hz. Of course, when sampling handwriting with discrete, finite-accuracy digitizers (e.g., 0.002 cm resolution, 0.004 cm accuracy) (Meeks & Kuklinski, 1990), higher sampling frequencies (e.g., 100 Hz) are required, especially if time derivatives have to be estimated (e.g., Teulings & Maarse, 1984). It will appear that many more intelligent computational models employ also 4 parameters per stroke and seem not more parsimonious than this unintelligent computational model. Some models use even more than 4 parameters, which is not parsimonious, and seem to perform a curve fit. The migration of the computational models runs from the original models, generating horizontal and vertical force components, via orientation-free models, mass-spring models, to symbolical and biomechanical models.

The average number of parameter updates per ballistic stroke provides a measure for the parsimony of a model. The initial parameter settings are not counted if they hold for all handwriting patterns. Two properties of a model may yield a further reduction of the "effective number of parameters", although this has not been applied to the models for reasons of comparability. The first property is the implementation of symbolic or hierarchical representations of movement units, e.g., allographs, or parts of them. For example, if the set of parameters, that generates an allograph, can be reused for each replication in all contexts, then the total number of parameters of a pattern equals the number of "allograph identifiers". The number of parameters for the actual allograph is merely a constant initial condition.

The second property yielding a reduction of the information contents of the set of parameters is the quantization of a parameter into a few discrete levels. Normally, parameters are scalars, having a finite number of "effective quantization" levels due to the limited accuracy of the handwriting motor system. The ratio of parameter quantization and effective quantization yields the reduction factor of the number of effective parameters of a model. The number of effective quantization levels in normal handwriting strokes can be derived from the speed-accuracy tradeoff according to Fitts' law. Fitts' law says that the duration of reciprocal aiming movements of amplitude A towards a target of width W increases linearly with the information measure $\log_2 (2A/W)$ in "bits". The factor of 2 expresses that A is actually the radius of the circle of diameter 2A within which all possible aiming movements of amplitude A are found. The duration increases with instructed accuracy according to 26 ms/bit and 43 ms/bit for the finger and wrist movements, respectively, in a wide range of amplitudes and accuracies (Langolf, Chaffin, & Foulke, 1976). This suggests that during a typical handwriting stroke of 140 ms the finger and wrist movements produce about 5.4 and 3.2 bits, corresponding to the number of effective quantization levels 2A/W of 40 and 10, respectively. It may be speculated that if both finger and wrist movements are controlled independently, each stroke contains about 8.6 bits, corresponding to 400 discrete strokes, which incidentally equals the empirical finding that a self-organizing net of strokes of at least 400 cells provides an adequate description of the handwriting strokes for automatic recognition (Schomaker & Teulings, 1990).

When taking also the different offsets into account, the speed-accuracy tradeoff relations for finger, wrist, and arm movements yield similar values of the relative accuracy, namely, for stroke durations of 140 - 200 ms 2.6 bits, corresponding to 6 effective quantization levels. The signal-to-noise ratio (SNR), yields similar values of the effective number of quantization levels in a more adequate task than the Fitts task, namely in a real handwriting task, where stroke durations are about 140 ms. The SNR is the ratio of the standard deviations of the movement amplitude sd(signal), and that of the movement noise sd(noise) (See Microscopic Models). The relation between the strict sizes A and W and the standard deviations is exactly A = 2sd(signal) and approximately W = 4sd(noise), namely, a Gauss-shaped probability distribution between + and -2sd(noise) corresponds to 95% hits within target W. Therefore, the number of effective quantization levels per component in a real handwriting task is 2A/W = SNR. The observed SNRs of about 6 in patterns of the vertical stroke sizes (Teulings, Thomassen, & Van Galen, 1986) suggests a lower number of effective quantization levels per component, which seems understandable as handwriting is more complex than reciprocal movements.

4.2 X and Y Force Models

One of the earliest characterizations of handwriting movements have been presented by Denier van der Gon and Thuring (1965). Their statements are interesting in that they provided a simple representation of the pen-point movement in handwriting. They suggested that the ratios of the stroke lengths are programmed by the duration ratios of the accelerative force bursts whereas overall size is programmed by force amplitude. The pen-paper and internal frictional forces were supposed to be negligible relative to inertial or accelerating forces. Finally, they suggested that movements are programmed in terms of separate horizontal and vertical components. This model has inspired many computational models of handwriting. Denier van der Gon, Thuring, and Strackee, (1962) used analogue force generators with fixed force rises up to a fixed amplitude. Vredenbregt and Koster (1971) have successfully regenerated handwriting. They used electromotors for horizontal and vertical movement components and manually adjusted the start and stop moments of fixed-amplitude currents (or forces or accelerations). They found that changes of the vertical component distorted the generated writing trajectory about as much as twice as big changes of the horizontal component. Under the condition of fixed force amplitudes for each of the two sets of agonists and antagonists involved, each stroke requires 4 parameters: The stop moment of the agonist force causing the acceleration, and the start moment of the antagonistic force, causing the decelerating force, respectively, for horizontal and vertical components independently. The agonist force has already started in the previous stroke and the antagonist force will stop in the next stroke, so that these parameters are not reckoned to the current stroke. If the force amplitude per component would have been a parameter also, this would require two more parameters per stroke and the model would not be parsimonious anymore.

Dooijes (1983) presents an equally parsimonious computational model of handwriting by assuming that the accelerating and decelerating force amplitudes vary per stroke, but that the stop moment of the accelerating and the start moment of the decelerating force coincide. A "bang-bang" force pattern results. Furthermore, instead of orthogonal axes, oblique axes are used, derived from the subject's preferred hand and finger movements separately. The axes are moving at constant speed from left to right. The question arises whether more realistic generation models exist than this bang-bang force pattern, and which are still parsimonious.

MacDonald (1966) examined the shape of the acceleration bursts during handwriting. The finger movements produced rectangular acceleration functions, while the wrist movements produced rather trapezoid acceleration functions, and the lower arm movements rather triangular acceleration functions. These force patterns occurred only in the "stroke-controlled" or ballistic movements (i.e., strokes taking less than 250 ms, having a single acceleration and a single deceleration phase). However, "continuously-controlled" movements (i.e., slower strokes) are produced at such a low rate that they show multiple acceleration phases per velocity phase (e.g., Maarse, Meulenbroek, Teulings, & Thomassen, 1987). Finally, extra force bursts occur when the pen tip suddenly "sticks" to the paper ("static friction" or "Coulomb friction"), which occurs where movement direction reverses such as in cursive allographs "c" or "u". This frictional force was hard to describe, but Denier van der Gon and Thuring (1965) suggested that friction may be neglected with respect to the accelerating or inertial force. Viscous force, which increases with velocity, appeared also negligible with respect to the inertial force: $k = \pi m/T < 0.1$, where T is the stroke duration, and m the effective mass of the hand.

Plamondon and Maarse (1989) compared the accuracy of reconstruction of the writing trajectories generated by several of these models and variants of them. They showed that many of the models can be represented by second-order or third-order transmission systems. The order of a system is the highest time derivative needed to relate the input and the output signal. The zeroth, first, or second time derivatives of the input represent the input signal itself, the rate of change of the input signal, and the acceleration of the input signal, respectively. To estimate the order the nerve-muscle interface and a muscle-pen (or hand-paper) interface were described separately.

The nerve-muscle interface can be derived from Hill's relation (e.g., Dijkstra, Denier van der Gon, Blangé, Karemaker, Kramer, 1973) and expresses the muscle-force output F(t) as a function time, the relative nerve activation level g(t) (ranging from 0 to 1), the muscle's maximal isometric force F_{max}, its shortening speed v(t), and its maximum shortening speed v_{max}:

$$F(t) = \{ ((F_{max}+a)*v_{max}) / (v(t)+v_{max}) - a \} * g(t),$$

where a is the muscle constant. For the forefinger muscles, vmax corresponds to pen speeds of 20 cm/s, whereas the highest pen speeds in normal-size handwriting are about 10 cm/s, so that v(t) << vmax. Furthermore, the minimum activation and deactivation times of the system are about 20 ms, whereas normal handwriting strokes have durations of at least 100 ms. Therefore, handwriting movements are well within the range of the physiological muscle specifications, so that $F(t) \approx F_{max}*g(t)$ (Plamondon & Maarse, 1989). This forms a zeroth-order transmission system. Furthermore, the nerve activation level g(t) is assumed to depend only upon the first and the zeroth time derivatives of the nerve firing rate so that the nerve-muscle interface is probably a first-order response system to the neural firing rate.

The muscle-pen interface can be described by a damped spring-mass system:

$$F(t) = m*a(t) + fv*v(t) + k*r(t) + f*Fz*|v(t)|$$

where r(t) is the off-equilibrium distance and v(t) and a(t), its first and second time derivatives, or velocity and acceleration, respectively, and |v(t)| the speed irrespective direction. Fz is the pen-paper pressure and m, fv, k, and f are the equivalent mass, the viscous friction constant, the muscle stiffness and the pen-paper friction, respectively (See also Wing, 1978; Dooijes, 1983). As the second time derivative (i.e., the acceleration a(t)) is the highest time derivative of position, this is a second-order transmission system. The order of the total transmission system is therefore three. However, if the nervous system is anticipating the first-order internal transformation, the order of the total system may be as low as two. So the system order of the models defined in the acceleration domain, require two integrations and is at least two and those defined in the velocity domain require only one integration so that the system order is at least one. The order had to be augmented by one if the shape of the pattern was a symmetrical trapezoid, a symmetrical triangle, or an exponential function (containing one adjustable parameter) or by two if it was Gaussian or sinusoid (containing two parameters). Examples of second-order systems are Denier van der Gon et al. (1962, 1965), Eden (1962), Mermelstein and Eden (1964), Hollerbach (1981), and Dooijes (1983). They consist of component movements along two axes, without friction or elasticity and two or three acceleration levels (namely, positive, zero, negative). Examples of third-order systems are MacDonald (1966) using trapezoid acceleration patterns and Yasuhara (1975) incorporating exponential acceleration patterns and some internal friction.

These and several other models were compared in terms of the spatial error between original and regenerated handwriting pattern, defined as the average distance relative to the stroke length. More precisely, the spatial error was calculated by the average of the surfaces between the recorded and the regenerated strokes, divided by their squared lengths. To regenerate a handwriting pattern, the effective accelerations and durations of the positive and negative phases were estimated for the horizontal and the vertical acceleration components separately.

These phases represent force bursts in horizontal and vertical directions, respectively. Analogously, the phases of the velocity components were estimated on behalf of the velocity-domain models. These phases represent "ballistic component strokes" as the components were analyzed separately. The shapes of the positive and negative phases could be approximated by rectangular, symmetrical trapezoid, symmetrical triangular, half sinusoid, Gaussian, or exponential rise-and-decay time functions (See Figure 7). The number of phases per component equals approximately the number of ballistic strokes. Therefore, 4 parameters per ballistic stroke are required: the durations and the effective accelerations or velocities for the horizontal and vertical components.

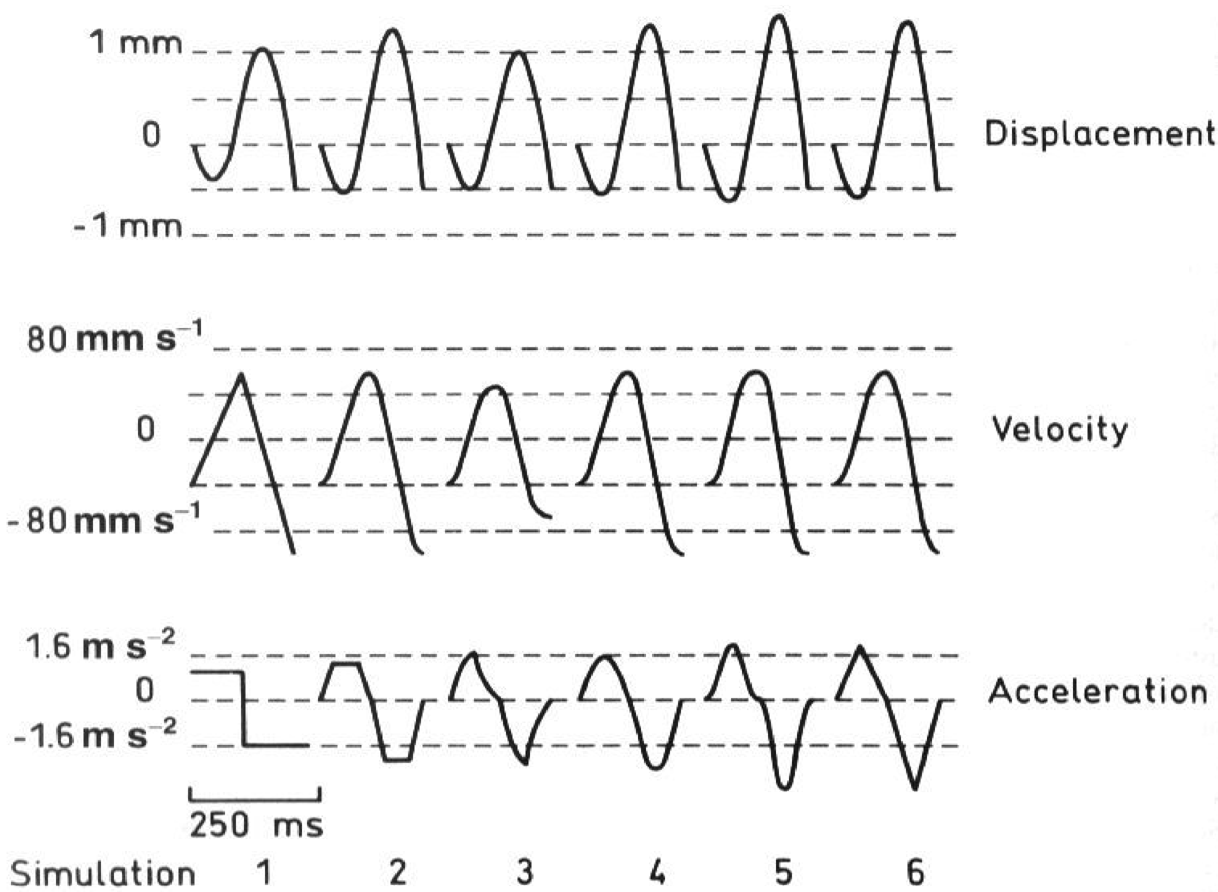

Figure 10.7: Some fitted time functions per stroke in acceleration domain with:1, rectangular; 2, summerical trapezoid; 3, exponential rise and decay; 4, half-sinusoid; 5, Gaussian; and 6 symmetrical triangular shape, and their shapes in velocity and displacement domains. [From Plamondon and Maarse, 1989.]

It appeared that most models accurately reproduced the handwriting patterns. The models defined in the acceleration domain were slightly worse than those defined in the velocity domain. Small spatial errors were achieved by approximating the acceleration components by rectangular or trapezoid functions, or by approximating the velocity components by several reasonable patterns, such as sinusoidal, Gaussian, or triangular time functions. There was no difference in accuracy between models of different system orders.

4.3 Orientation-Free Force Models

There is a class of models, which do not assume any specific x or y axes, or axes related to biomechanical joints. Morasso and Mussa Ivaldi (1982) and Morasso, Mussa Ivaldi, and Ruggiero (1983) state that movement patterns are unlikely to be described in terms of joint angles but rather in some spatial reference frame, which does not need to be the body-oriented horizontal coordinates. They assumed that discrete "circular strokes" are generated in a discontinuous process while the inertia of the finger-hand-arm system smoothes the movement by acting as a low-pass filter (e.g., Van Galen & Schomaker, 1991) (See Figure 8). Circular strokes are defined as: (1) circle segments of specific curve radius, arc length, and orientation, such that it is tangential to the trace at the point of peak velocity or inflection, (2) having a half phase of a Gaussian velocity function during the interval from the previous absolute-velocity peak to the current peak and the other half phase from the current peak to next peak, (3) each half overlapping in time with the previous and the following circular strokes, respectively. Overlapping circular strokes are averaged sample-by-sample. Therefore, 6 parameters per ballistic stroke are needed: its curve radius, arc length, orientation, peak velocity, duration between previous and following velocity peak, and the asymmetric position of the peak velocity between the previous and following ones. The accuracy of this model does not seem as high as the simpler models, which were based on component movements per axis (Plamondon & Maarse, 1989).

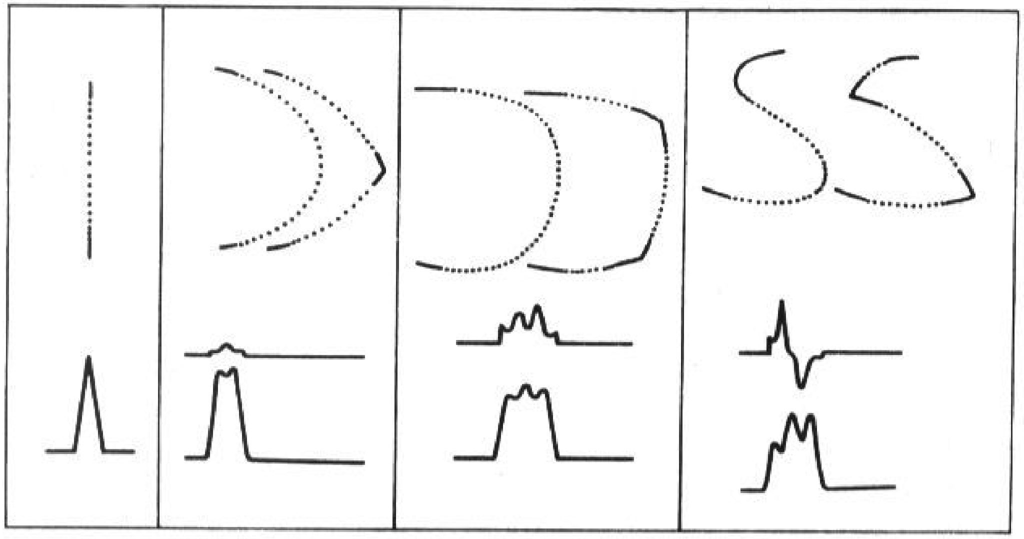

Figure 10.8: Examples of the performance of the circular-stroke composition model for four movement patterns in the four frames. Dots are equally spaced in time. Each frame shows the simulated movement pattern (top left), the underlying circular strokes with Gaussian velocity time functions (top right), and the curvature and absolute velocity time functions (bottom). [From Morasso, Mussa Ivaldi and Ruggiero, 1983.]

In another paper Plamondon (1989) provides a model which allows a more accurate reconstruction of writing patterns. Like in the previous model, absolute velocity is approximated by a piece wise Gaussian function but, instead of the previous constant-radius approximation and the concept of overlap, the angular velocity is also approximated by a piece-wise Gaussian function. Thus, again two parallel motor signals are required, but they do not refer to Cartesian axes. The piece-wise Gaussian functions are generated by a hypothetical velocity nerve signal, consisting of a sequence of rectangular time functions (Plamondon, 1989). The height of each block represents the gain of the muscular speed-generator. In order to simulate sharp movement reversals, discontinuities had to be inserted in the angular velocity. The cursive-script word "bug", containing about 12 ballistic strokes, could be reconstructed very accurately, using 60 parameters for the tangential velocity and 52 parameters for the angular velocity. Therefore, as many as 9 parameters per ballistic stroke were used, which is much more than the minimum of 4 parameters per stroke.

4.4 Mass-Spring Model

The mass-spring model was introduced to model fluent movement trajectories in a parsimonious way. Hollerbach (1981) assumed a horizontal (x) axis, parallel to the left-to-right translation and a vertical (y) axis, and fitted piece wise sinusoids as if they resulted from a frictionless mass-spring system. For the vertical movement component, the mass consists of the mass of the pen and the fingers whereas for the horizontal movement component, the mass consists of the mass of the whole hand and the pen. Various sinusoids can be concatenated to a fluent handwriting trajectory, where the equilibrium points, the stiffness, and some initial conditions of the mass-spring system are changed at specific moments. The model would generate repetitive loops such as "eeee" very parsimoniously as no parameters need to be changed during this pattern. The model is expressed in terms of the velocity time functions Vx(t) and Vy(t), respectively. Velocity is used for convenience instead of position or acceleration, which would also be sinusoids.

$$Vx(t) = Vx\_peak*\sin(\omega x*t + \varphi x) + c$$

$$Vy(t) = Vy\_peak*\sin\varphi y*t + \varphi y)$$

where Vx_peak and Vy_peak are the velocity amplitudes, φx and φy the angular frequencies, or frequencies multiplied by 2π, and φx and φy the initial phases, respectively. The constant-velocity, horizontal left-to-right movement c is added to the horizontal component.

Some simplifications can be made. Of course, only φx - φy = φ is relevant as the initial phase can be set in the initial condition. Furthermore, the basic setting of the peak velocities in both horizontal and vertical directions can be chosen equal to twice the horizontal left-to-right velocity, i.e., Vx_peak = Vy_peak = 2c. So repetitive loops are generated by circular movements, while the centre of the circle moves with half the circling speed. Finally, the horizontal and vertical frequencies may be chosen equal, i.e., φx = φy = φ. Unequal frequencies would only be needed in patterns like "8". Various basic patterns can be generated by modulating φ: upright loops (φ = 90 degrees), slanted loops (φ = 60 degrees), guirlands (φ = 30 degrees), waves (φ = 0 degrees), and arcades (φ = -30 degrees). Another important parameter to influence stroke shape is the horizontal velocity Vx at the moments where Vy changes from upward to downward (i.e., at the top of a stroke): Vx = c - Vx_peak*sin(φ). So if a sharp movement reversal has to be programmed such as at the top of allograph "c", requiring Vx to change sign, the parameters φ, Vx_peak or c may be adjusted simultaneously.

Ascenders and descenders can be generated by increasing Vy_peak and by decreasing φ, similar to the observed change of both force and duration in meso context. In order to maintain a stable baseline Vy_peak should only be changed at the segmentation points where the upward movement turns into a downward movement or vice versa, i.e., Vy=0. These upper and lower segmentation points correspond with the boundaries of ballistic strokes, i.e., points of minimum absolute velocities. An ascender can be generated at a lower segmentation point and a descender can be generated at an upper segmentation point. However, if only Vy_peak is adjusted, between "e" and "l", then slant would change as well, so that at least two parameters have to be adjusted simultaneously. The second parameter to be changed follows from the expression of the slant, which was defined here by the complementary angle needed to shear the writing trace so that slanted loops obtain vertical-mirror symmetry. This measure appeared to fit well the slant perceived by subjects (See also Maarse & Thomassen, 1983). The slant β in terms of model parameters equals: β = arctan (Vy_peak / (Vx_peak*cos(φ))), which again depends upon one or more parameters. When generating an ascender, Vy_peak was to be increased, so that either ø or Vx_peak have to be changed as well, in order to maintain slant.

Another way to vary stroke length in meso context is to decrease only angular frequency ω, which does not affect slant and may seem more suitable. However, observations of size variation in meso context show that both duration and force are varied (See Microscopic Models). This suggests adjustment of both ω and Vy_peak, respectively. As several model parameters have to be changed simultaneously, it is not easy to suppose that these parameters form a realistic representation in the graphic motor-patter code. Other functions could have been used instead of sinusoid velocity functions, e.g., triangular functions, which correspond with rectangular functions in the acceleration domain (e.g., Plamondon & Maarse, 1989), yielding: β = arctan (Vy_peak / Vx_peak), which contains only an amplitude ratio and which would involve less parameters. If trapezoid functions would have been used in the acceleration domain, the latter still holds but for a limited range of phase φ.

An example of a simulation of the letter sequence "elye", counting 11 ballistic strokes, requires 30 parameter changes and time moments and 10 initial parameter settings, which may be pattern dependent. This yields only 3.6 parameters per stroke, which is parsimonious indeed, as it is less than 4 (See Figure 9). Although the model would generate repetitive loops parsimoniously, it appeared difficult to generate more complex repetitive patterns like "ellellell...". Furthermore, normal handwriting does not have a predominant sinusoidal movement (Teulings & Maarse, 1984), nor a constant left-to-right trend (e.g., Thomassen & Teulings, 1983a), because in that case there would not have been any problem to satisfy the two-third power law in micro context (See Microscopic Models).

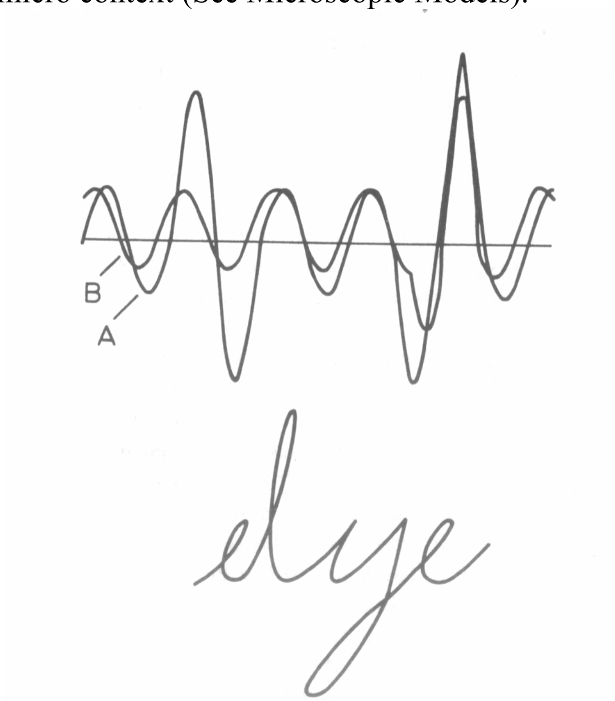

Figure 10.9: Vertical (A) and horizontal (B) velocity components generated by the mass-spring model and the resultant writing trajectory. [From Hollerbach, 1981.]

4.5 Symbolical and Biomechanical Models

There is still another motor principle than the principle of a mass-spring system: minimizing the mean squared rate of force change. In the frictionless case (and when reducing the writing apparatus to a single-joint stick-and-point-mass system) the rate of force change can be approximated by the third time derivative of the horizontal and vertical position time functions, which is known as jerk. This appears to generate realistic, smooth aiming movement patterns (Edelman & Flash, 1987; Flash & Hogan, 1987; Nelson, 1983). For example, in straight movements the minimum-jerk model generates Gaussian-shaped lines with Gaussian velocity functions having peak velocities $V\_max = 1.875s/T$, where s equals the stroke size and T the stroke duration. Edelman and Flash (1987) suppose in their "minimum-jerk model" that cursive script consists of a sequence of curved segments. For curved segments the minimum-jerk model needed to be extended by adding a "via point" near the point of maximum curvature (Flash & Hogan, 1987). Such a curved segment will be called here a "via stroke". A via stroke corresponds to a pair of "ballistic strokes". The solution of the minimization of the mean squared jerk with a via-point constraint are 5th-order spline functions for the x and y coordinates separately. For example, the x coordinate as a function of time t is:

$$x(t) = ax0 + ax1*t + ax2*t^2 + ... + ax5*t^5 + px*\max(0,t-t1)^5,$$

where ax0 ... ax5 and px are the spline parameters and t1 is the optimized time parameter to reach the via point. The minimum-jerk model shows that reconstruction of the handwriting movements on the basis of "kinematics from shape" matches experimental data well. E.g., curved via strokes obtain bi-modal velocity patterns.

Edelman and Flash (1987) tried to simulate cursive-script patterns and found that a limited set of basic via strokes is sufficient to represent cursive script: hook (like cursive "i" without dot), cup (like cursive "v"), gamma (like cursive "l"), and oval (like cursive "o") (See Figure 10.). These via strokes can be classified on the basis of the configuration of the beginning, via and endpoints. The form of the basic via strokes are invariant under rotation, translation, and scale. There were a few discrete parameters: Each via stroke was either small or big. Furthermore, vertical velocity may or may not change direction at the via point, yielding retrograde or regular via strokes, respectively. Finally, vertical position of the via point relative to the baseline and letter-body size may either be: upper, middle, or lower. Variability of fast and sloppy handwriting is considered as random perturbation of the movement parameters. Using this symbolic description, it is, in principle, possible to generate novel letter sequences in a particular handwriting, or to automatically recognize on-line handwriting (Edelman, Flash, & Ullman, ,1990).

Figure 10.10: The four basic via strokes: hook, cup, gamma and oval, respectively. All cursive characters can be represented as combination of rotated, translated and scaled versions of these via strokes. [From Edelman and Flash, 1987.]

However, the minimum-jerk model with only via-point constraints does only allow simulation of the hook. While trying to generate the other three basic via strokes it appeared that additional via points did not provide a solution as it is unclear which other via point to select than the natural one near the point of maximum curvature. Instead, other constraints should be added, which should satisfy the kinematics-from-shape principle: the direction (i.e., dy/dx) at the beginning and endpoints of the via stroke. Because of the border condition of zero velocities and accelerations, the direction is undefined, so that according to a mathematical rule, the direction of the lowest nonzero derivative has to be used, which turns out to be jerk. But when jerk needs to be constrained, it does not make sense to minimize mean squared jerk. Instead, the mean squared fourth time derivative (i.e., snap) should be minimized. This yields a 7th-order spline function, analogous to the 5th-order spline function above. The "minimum-snap model" requires still a via point, as without via point the accuracy turns out poor.

The minimum-snap model employs 18 parameters per via stroke: ax0, ... ax7, px, ay0, ..., t1). However, most parameters, and even duration t1 till the via point, are fixed by the boundary conditions. The boundary conditions are that beginning and end of the via strokes should have a continuous and smooth connection to the adjacent via strokes, i.e., equal horizontal and vertical positions, velocities, accelerations, and jerks. Furthermore, two boundary conditions from the via point yielding 8 fixed parameters per via stroke. The two minimization equations for x(t) and y(t) per via stroke fix two more parameters. Therefore, 6 parameters per via stroke, i.e., only 3 parameters per ballistic stroke are required: x and y positions of beginning and via points, direction and amplitude of the jerk at the beginning point, and the mean absolute jerk.

Wann, Nimmo-Smith, and Wing, (1988) showed that the minimum-jerk model generates movements which satisfy the two-third power law (Lacquaniti, Terzuolo, & Viviani, 1983; See Microscopic Models). However, both models generate symmetrical velocity functions, which are not observed in movements which are slower than maximally fast. In slower movements, velocity patterns are more skewed with peak velocity occurring earlier than halfway. Wann et al. wondered how the motor system, which can sense only joint angle, stretch and the rate of stretch, is able to minimize jerk at all. They suggested extending the minimum-jerk model by a psychological model, without claiming its biomechanical validity, where an inertial mass is suspended with springs, having a certain stiffness inside the visco-elastic limb (See Figure 11.). The extent of off balance and its rate of change allow detection of acceleration and its rate of change (i.e., jerk), respectively. Furthermore, if it is assumed that instead of the limb the suspended mass would be moving according to the minimum-jerk model, then the sum of the jerk of the limb plus the jerk of the mass relative to the limb has to be minimized. Simulations at low speeds, where stiffness is low, confirm that this extended minimum-jerk model generates the skewed velocity patterns which are in accordance with empirical data. Recently, Plamondon (1991) provided another explanation for the skewed velocity profiles by assuming a set of independent (parallel) subsystems, each producing an arbitrary probability density of a velocity command. The net result is a probability-density which is the product of all these component probability densities. Therefore, its logarithm is the sum of a number of arbitrary probability densities, which iterates to a log-normal probability density for the velocity signal of an aiming movement. This generates also an asymmetric velocity pattern.

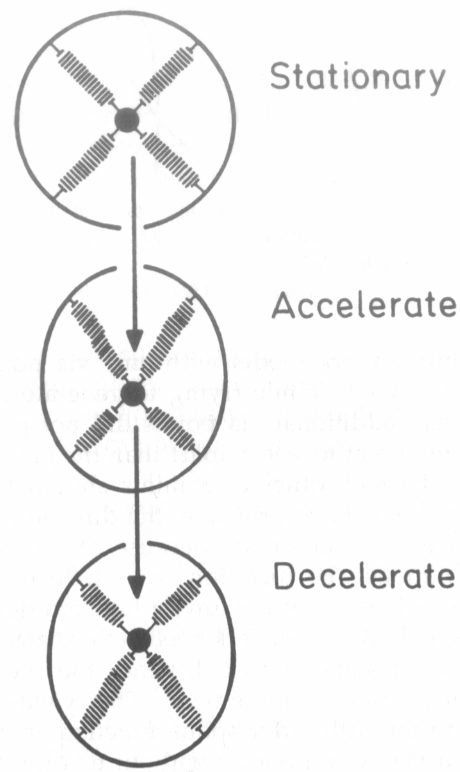

Figure 10.11: Representation of a viscoelastic model which attempts to extend the minimum jerk model. The mass inside the body (e.g. the arm) is suspended by surrounding tissues, represented by damped springs. As the body accelerates, the distortions of the surrounding tissues provide information about direction and extent of the acceleration and its rate of change, i.e. jerk. In this model, the mean squared jerk of the mass is minimized, instead of that of the outside of the body. [From Wann, Nimmo-Smith and Wing, 1988]

A fully symbolic model to simulate handwriting has been presented by Schomaker, Thomassen, and Teulings, (1989). Input to the model is an arbitrary text in terms of a sequence of allographs, analogous to the allographic buffer. The outputs are cursive-script movements of the writer's own style of handwriting. Interesting features of this model are the implementation of visual feedback, and the writer-specific motor memory, analogous to the graphic motor-pattern store ("symbolic letter descriptions"). Visual feedback monitors, in fact, only the baseline and lineation levels. It is implemented by an exponentially decaying lineation memory, which adjusts vertical sizes of subsequent strokes after an inappropriate stroke size causes a departure from the imaginary baseline and lineation levels.

The symbolic letter descriptions of the allographs were trained on the basis of a corpus of a writer's handwriting. Thus, each allograph is represented by a sequence of strokes, where each stroke is represented by 4 stroke parameters: dX (horizontal displacement per stroke), dY (vertical displacement per stroke), T (compound stroke duration), and C (stroke-shape factor). Pen status may form a fifth feature but in cursive-script it is more parsimonious to insert (penup) and (pendown) commands at appropriate times. In the previous models pen status was not included. These parameters have been based upon the average of several manually selected replications of the same allograph in various contexts. Averaging is allowed as context effects are only marginal, especially for spatial features (Thomassen & Schomaker, 1986; Schomaker & Thomassen, 1986). The strokes of an allograph appear indeed to be retrieved as complete units (Teulings, Thomassen, & Van Galen, 1983). At the symbolic module the spatial features have been quantized: dX/dY could be: close, normal, or far. dY could be: descender, base, body, or ascender. However, in natural handwriting, quantization of the vertical positions of the beginning and end of a stroke may be more fine grained than 4 levels: Therefore, each level was split up into 2 or 3 sub-levels (e.g., body-plus, body, body-minus, etc., but ascender-plus and descender-minus were omitted), yielding a total of 10 lineation levels for vertical position.

Although there is no evidence that relative durations are represented in motor memory (Teulings et al., 1986), the "compound stroke duration" T is used as a parameter to conveniently interpret the required "stroke-shape factor" C. Namely, the interval between successive zero crossings in the X velocity component (i.e., t1 (vx=0) and t2 (vx=0), respectively) can be defined as the X-stroke duration, and similarly for the Y-stroke duration. The compound stroke duration is defined as the average of the X and the Y-stroke durations:

$$T = \{ (t2(vx=0) - t1(vx=0)) + (t2(vy=0) - t1(vy=0)) \} / 2$$

so that T ranges between about 50 and 150 ms. The stroke-shape factor C is defined as the time interval between two nearby zero crossings of X and Y velocities, relative to the compound stroke duration T:

$$C = (t1 (vx=0) - t1 (vy=0)) / T.$$

The shape factor is a generalized phase difference between X and Y velocity time functions. If the X velocity component is ahead of the Y component, then the stroke shape will form (part of) a counterclockwise loop (i.e., $-1.5 < C < 0$). In the opposite case, the stroke shape will form (part of) a clockwise loop (i.e., $0 < C < 1.5$). In the special case that the X and Y zero crossings occur simultaneously ($C = 0$), there will be a sharp stroke ending, followed by a movement reversal (See Figure 12).

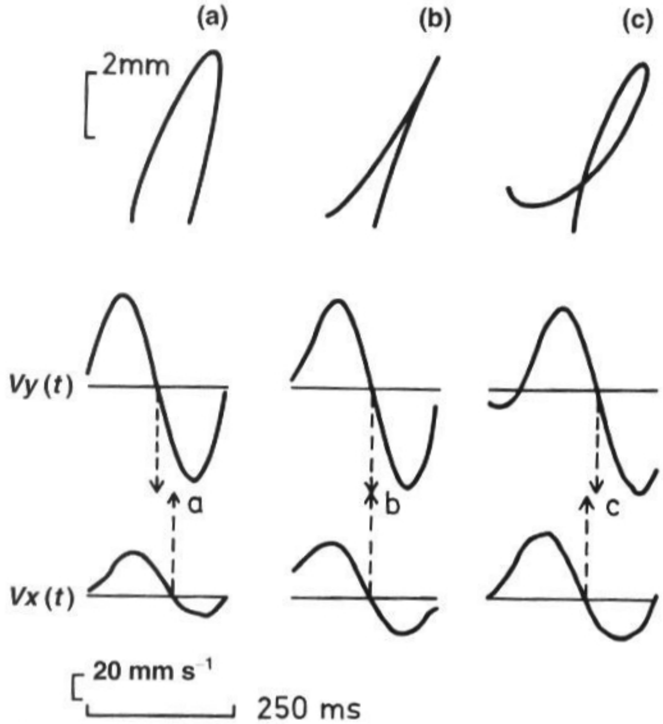

Figure 10.12: Basic stroke shapes and their relative timing in the velocity domain. (a) Blunt, clockwise stroke transition, where the zero crossing of the horizontal velocity occurs after that of the vertical velocity, yielding shape factor $C > 0$. (b) Sharp stroke ending, where both zero crossings occur simultaneously, yielding shape factor $C = 0$. (c) Counterclockwise looping stroke transition, where the zero crossing of the horizontal velocity occurs before that of the vertical velocity, yielding shape factor $C<0$. [From Schomaker, Thomassen and Teulings, 1989.]

The procedure to translate the allographic-code input into the required movement patterns is as follows. In the symbolic module, specific connecting strokes have to be inserted between pairs of allographs and punctuation signs. The connecting stroke depends upon the final stroke of the preceding allograph and the initial stroke of the subsequent allograph. The parameters of the connecting stroke have been estimated by the average of the replications in similar contexts in the corpus of handwriting. For example, the connecting stroke in "me" is assumed similar to the one in "ne". The "cursive connections grammar" contains the generic rules prescribing the connecting strokes to be inserted. For example, the input "an ad..." is expanded to: "(pendown) (a) (base to midline, clockwise, close progression) (n) (base to base-plus, sharp ending, close progression) (penup) (space) (pendown) (a) (base to midline, sharp ending, normal progression) (d) ...".

Subsequently, at the quantitative level, the strokes per allograph are selected from a sort of graphic motor-pattern store. The "quantitative letter descriptors", describe the strokes in terms of dX, dY, T and C. The compound stroke duration T and the form factor C allow the approximation of the moments in time where X and Y velocities change sign. In order to generate the kinematics of a handwriting trajectory a general form of the velocity pattern is selected. For convenience, a sinusoid velocity time function is selected to fit between successive zero crossings of X and Y velocities, which appeared to approximate handwriting movement patterns relatively well (Plamondon & Maarse, 1989).

Although only 4 parameters per stroke are used to generate handwriting patterns, the effective number of parameters is less. Namely, certain stroke sequences, representing allographs, can be retrieved by just one parameter: the allograph identifier. Furthermore, dY is quantized. Interesting is that the generated handwriting patterns do not form a curve fit of a particular pattern, but appear as noisy as individual replication of the same pattern, while showing the personal traits of the original writer. Also in reality, a writer cannot reproduce a writing pattern exactly (e.g., Teulings et al., 1986). Analogously to assessing the typewriting model in Rumelhart and Norman (1982), a quality measure is needed, which tells to what extent the generated pattern is within the natural variations of the writer. The correlations of the horizontal and vertical stroke sizes and durations between the original patterns appeared of the same order as those between the simulated pattern and the original ones. Therefore, the simulated pattern fits well within the set of the original patterns of handwriting so that the present simulation model is sufficiently accurate.

4.6 Summary

Many other computational models for specific purposes such as automatic on-line handwriting recognition and signature verification exist. Appropriate models are attempting to describe the handwriting pattern in a parsimonious set of parameters, which may suggest that these parameters are less sensitive to motor noise and primarily express the underlying movement information. However, in on-line handwriting-recognition systems, a redundant set of higher- and lower-level features may have advantages, as the self-learning expert system may be able to select the statistically relevant features itself. It appears that many models require the theoretical minimum of about 4 parameters per stroke. Models with more emphasis on curve fitting may require more parameters. However, the accurate regeneration of writing patterns does not need to be the final target, since the motor system does not allow the accurate reproduction a movement pattern either. Therefore, the symbolic models seem attractive as they may show the same kind of noise when reproducing a handwriting pattern as if a writer were reproducing the pattern.

5 CONCLUSION

This chapter presented several theories on handwriting production, ranging from motor-pattern representation and retrieval to trajectory formation. It was intended to describe the theories in common terms, in order to proceed in the direction of the unification of motor theories, and particularly of handwriting-motor theories, and related skills such as typewriting, sign language, and speech. Eventually, this may lead to application of this motor knowledge in more remote domains of motor skills, such as grasping, posture, gait, jumping, and navigation. Various theories of different scope support the notion of modularity of the motor system. Understanding the motor system of handwriting may lead to a well-founded top-down approach for various important questions in handwriting research. They include the following:

(1) How can the body of motor knowledge presented here, be integrated into the world of motor control?
(2) What developmental paths are followed in the various modules as a function of maturation, exercise, aging, or nerve diseases?
(3) Can changes in the generation model, be it theoretical, procedural, empirically based, or neurally inspired, simulate the above changes in motor behavior?
(4) What automatically estimated motor characteristics can serve as measures of "neurological fitness" in child development, or as a precursor for a nerve disease?
(5) What do children learn during handwriting instruction and can immediate feedback via computer-assisted instruction help?
(6) Which features of the allograph shapes are appropriate for automatic handwriting recognition, and for reconstruction of the intended motor programs?
(7) Which features of the motor programs and the translation processes are subject-specific so that they may serve for writer identification and verification in handwriting expertise?

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