1. The improved Hollerbach model
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One of the first, if not the first, oscillatory model of handwriting was proposed by Hollerbach [1981]. It comes jointly with a spring-mass model of the arm apparatus, which will not be presented here.

In this model, handwriting is seen as the result of two superimposed oscillators on two distinct directions of the plane. Although any non-sinusoidal oscillators could work as well, it is more convenient to use sinusoids. Moreover, the choice was more compliant with the spring muscle model. Oscillators time evolution is defined as:

 $\frac{dx}{dt}=asin(ω\_{x}t+ϕ\_{x})+c$ (1)

 $\frac{dy}{dt}=bsin(ω\_{y}t+ϕ\_{y})$ (2)

where *𝑎* and *𝑏* are the horizontal and vertical velocity amplitudes, $ω\_{x}$, $ω\_{y}$, $ϕ\_{x}$ and $ϕ\_{y}$ are respectively the frequencies and the phases associated to these directions. *𝑐* represent the constant displacement to the right when writing. Direction on which oscillators vibrates are not necessarily chosen perpendiculars according to usual horizontal and vertical axis. It would be advocable to choose the horizontal axis for one of them and the slant direction for the other. In the rest of this paper we use the canonical direction of plan space as the directions of the two oscillators.

Model parameters (ie. $a$, $b$, $ω\_{x}$, $ω\_{y}$, $ϕ\_{x}$ and $ϕ\_{y}$) are supposed to be piecewise constant. All these parameters change when the vertical velocity is null.

Interestingly, the slant is described by the angle *𝛽* which can be expressed as :

 $tanβ=\frac{b}{acosϕ}whereϕ=ϕ\_{x}-ϕ\_{y}$ (3)

Another interesting value is the value of the horizontal velocity when the vertical velocity is null:

 $Ψ=\frac{dx}{dt}(t\_{y\_{0}})=c-asinϕ$ (4)

The sign and magnitude of this value indicates the particular shape of the trace at this point. If $Ψ$ is next to zero then the top corner will look sharp. If it is positive, the top corner will become rounded. Conversely, a negative value of $Psi$ will result in a full loop. This behavior is shown in table 1.

|  |  |
| --- | --- |
| Shape  |  $Ψ$  |
|   |  1.262  |
|   |  -43.74  |
|   |  37.41  |

Table 1: The shape of the top corner of the trace depends on the sign and magnitude of $Ψ$.

* 1. Improving the oscillatory model

 Hollerbach’s model does not address important questions:

* Is there a way to quickly extract parameters, given a recorded trace ?
* Is the ad-hoc *𝑐* parameter really necessary?
* Parameters (*𝑎*, *𝑏*, $ω\_{x}$, $ω\_{y}$, $Φ\_{x}$ and $Φ\_{y}$) are only allowed to change at times of zero-crossing vertical velocity: is there any reason for this dissymmetry between x and y axes ?

The main computational drawback of the Hollerbach model is the way we can get the parameters from the trace: it is a non linear curve fitting problem. Usual optimizations methods are costly; we will present an algorithm that is much more efficient. This efficiency is possible because of our choice (that will be presented later) of the moments when parameters are allowed to change. For a comparison between our method and usual optimization methods, see [André,submitted].

The c parameter is of course representing the constant drift of the hand from left to right when writing, but it seems a rather ad-hoc way for taking it into account. Moreover, it imposes the X axis to be exactly aligned with the trace direction, and it introduces a dissymmetry between X and Y. We will show that it can be simply omitted, and that the drift can be accounted for by cumulating phase values between cycles.

About the dissymmetry between X and Y, it is true that the majority of shape changes occur at Y velocity zero-values, which is visible in the fact that most loops and peaks are directed upwards or downwards. However, an allograph such as “k” has clearly a parameter change on the X velocity zero crossing near point a, which Hollerbach’s model cannot handle properly.

* 1. Our improved Hollerbach model

Our improved version of Hollerbach’s model is both simpler, symmetric and allows for a fast parameters extraction algorithm. It is completely described in the following paragraphs:

cinematic equations

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